Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 19

Time: 3 Hours. **Total Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A - E.
2. Section A has 18 multiple choice questions and 2 Assertion-Reason based questions carrying 1 mark each.
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.

Section A

Section A consists of 20 questions of 1 mark each.

Choose the correct answers to the questions from the given options. [20]

1. The LCM of 12, 15 and 21 is

A. 410
B. 420
C. 440
D. 450

2. Find the roots of $x^2 - 3x - 10$.

A. 5 and -2
B. -5 and 2
C. -5 and -2
D. 5 and 2

3. Find a quadratic polynomial with $\frac{1}{4}, -1$ as the sum and product of its zeroes respectively.

A. $k(4x^2 - x + 4) = 0$

B. $k(4x^2 + x - 4) = 0$

C. $k(4x^2 - x - 4) = 0$

D. $k(4x^2 + x + 4) = 0$

4. The lines $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ are

A. parallel

B. intersecting

C. coincident

D. none of above

5. For three terms p, s, q to be in A.P.,

A. $2p = p + s$

B. $p > s > q$

C. $p < s < q$

D. $2s = p + q$

6. If a point (c, d) lies in the 3rd quadrant, which of the following is true?

A. c is positive and d is negative

B. both c and d are positive

C. both c and d are negative

D. c is negative and d is positive

7. Find the distance between the points (0, 0) and (36, 15).

A. 29

B. 39

C. 49

D. 59

8. A circle can have ________ tangent/tangents.

A. one

B. two

C. four

D. infinite

9. Which of the following is not a test of similarity?

A. SSS

B. SAS

C. AAA

D. SSA

10. Corresponding _______ of similar triangles are equal.

A. Sides

B. Areas

C. Perimeters

D. Angles

11. If $\cot \theta = \frac{7}{8}$, then evaluate $\tan^2 \theta$.

A. $\frac{8}{7}$

B. $\frac{49}{8}$

C. $\frac{49}{64}$

D. $\frac{64}{49}$

12. If $\sin A = \frac{3}{4}$, calculate tan A.

A. $\frac{3}{\sqrt{2}}$

B. $\frac{3}{\sqrt{5}}$

C. $\frac{3}{\sqrt{7}}$

D. $\frac{3}{\sqrt{11}}$

13. Raju's teacher checked the solution paper of Raju and found zero errors. So she wrote the following:

"Congratulations Raju! The number of errors in your paper is equal to $\cos \theta$".

What will be the value of θ here?

A. 0°

B. 30°

C. 60°

D. 90°

14. Find the area of a sector of a circle with radius 6 cm, if angle of the sector is 60°.

A. $\frac{132}{3}$ cm^2

B. $\frac{132}{2}$ cm^2

C. $\frac{132}{7}$ cm^2

D. $\frac{132}{5}$ cm^2

15. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment.

A. 28.6 cm^2

B. 26.6 cm^2

C. 24.6 cm^2

D. 22.6 cm^2

16. Time taken by a group of swimmers for different range of ages is being recorded and maintained in a table. Which formula could be used to find the middle-most age?

A. $\frac{\sum fx}{\sum f}$

B. $a + \frac{\sum fd}{\sum f}$

C. $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

D. $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

17. Two coins are tossed simultaneously. Find the probability of getting at most one head.

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. 0

18. Find the mode.

Daily wages (in Rs.)	Number of workers (f_i)
100	12
120	14
140	8
160	6
180	10

A. 100
B. 120
C. 160
D. 180

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. **Statement A (Assertion)**: A spherical glass vessel has a cylindrical neck 7 cm long and 4 cm in diameter. The diameter of the spherical part is 21 cm. Hence the quantity of water it can hold is 4939 cm^3. Use = $\pi = \frac{22}{7}$.

Statement R (Reason): Quantity of water it can hold = volume of spherical glass vessel + volume of cylindrical neck

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

20. **Statement A (Assertion)**: If the solution of system of equations $x - y = 4$ and $x + y = 6$ is $x = p$ and $y = 2q$ then $p = 5$.

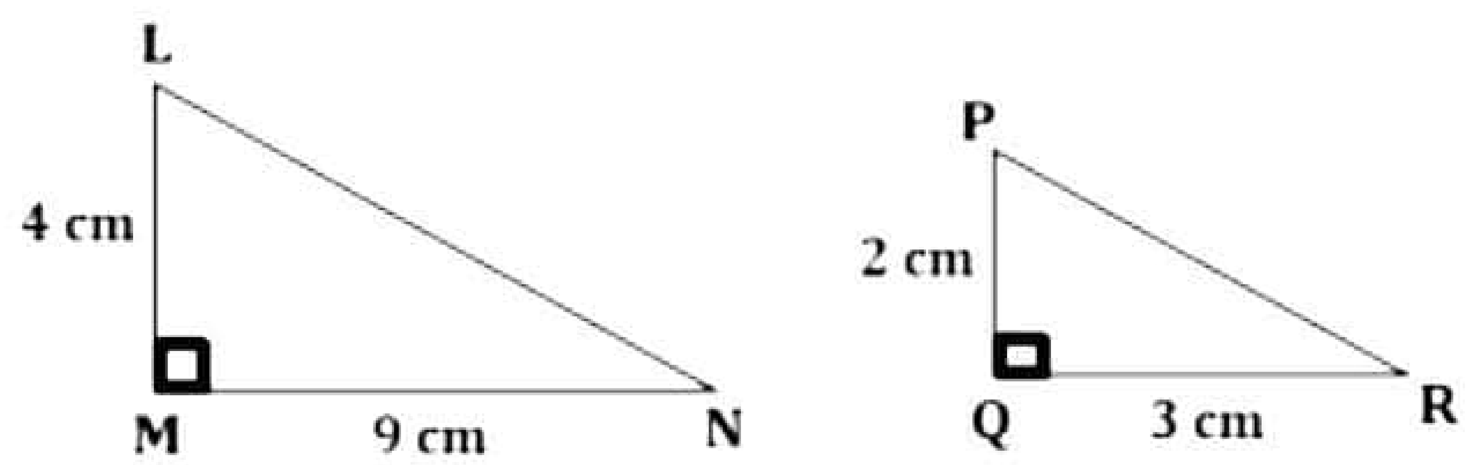
Statement R (Reason): A pair of linear equations in two variables can be algebraically solved by elimination method.

A. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
B. Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
C. Assertion (A) is true but reason (R) is false.
D. Assertion (A) is false but reason (R) is true.

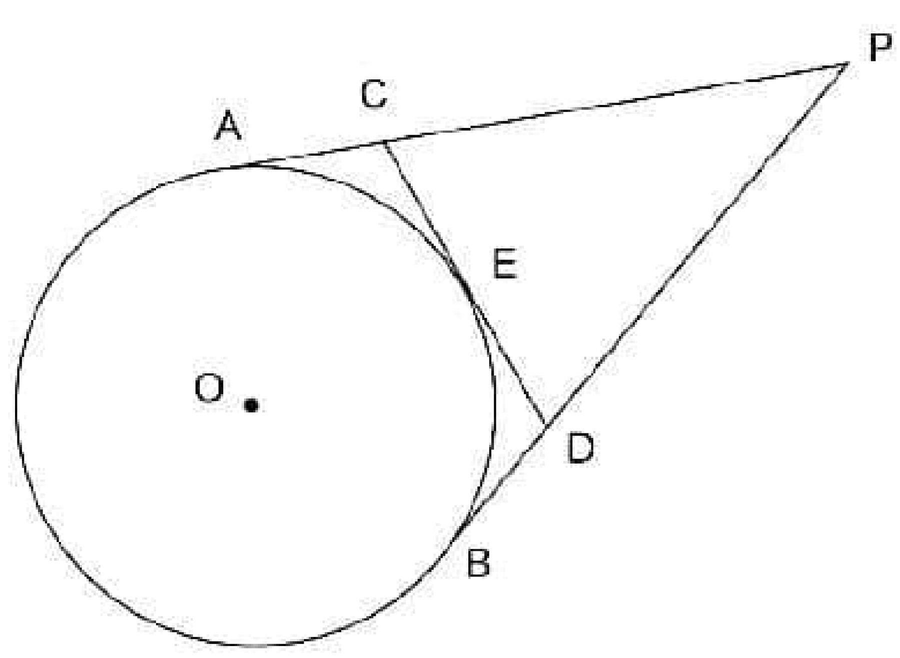
Section B

21. Check whether (4, −1) is a solution of system of equations $x - 2y = 6$ and $2x + 2y = 6$. [2]

22. Is $\Delta LMN \sim \Delta PQR$? [2]



23. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of ΔPCD. [2]



24. Show that the area of a sector is half the product of the length of an arc and radius. [2]

OR

A chord of a circle of radius 8 cm subtends a right angle at the centre. Find the area of the corresponding:

(i) Minor segment (ii) Major sector

25. Prove that $\sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}} = 2\operatorname{cosec} A$ [2]

OR

Prove that: $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \operatorname{cosec}\theta$

Section C

Section C consists of 6 questions of 3 marks each.

26. In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108, respectively. Find the minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject. [3]

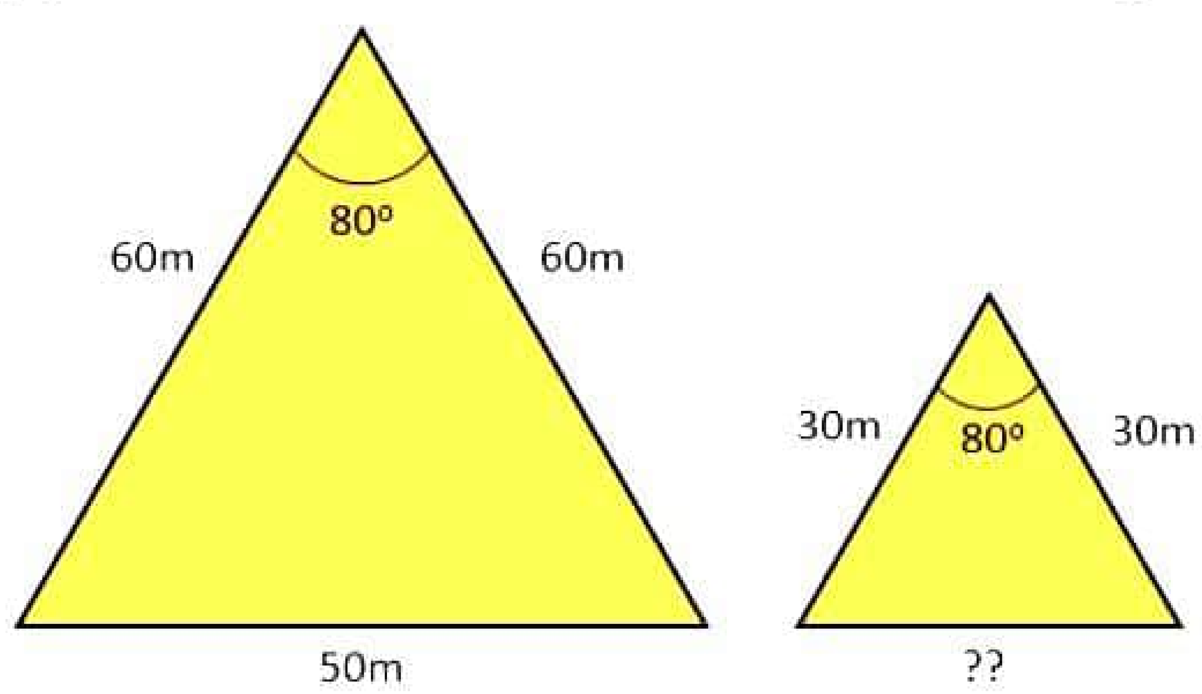
27. Verify the relationship between the zeroes and the coefficients of $t^2 - 15$. [3]

28. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Formulate the quadratic equation in terms of speed of the train. [3]

OR

Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

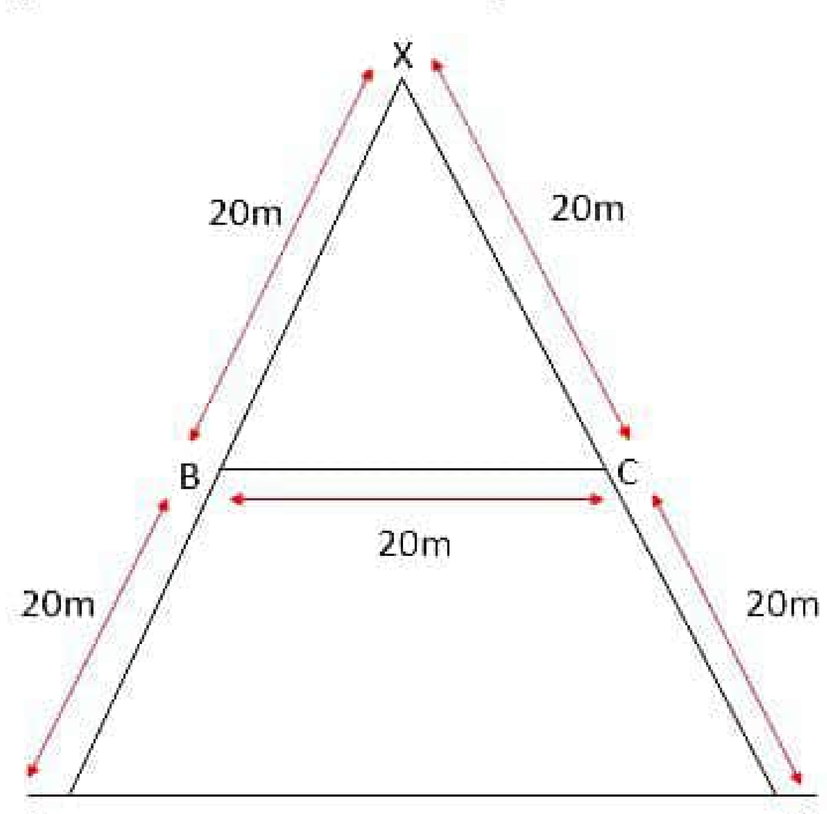
29. Atul went to Egypt to see the pyramids. Now the front side of two adjacent pyramids are as shown in the figure below. [3]



Calculate the base of the smaller pyramid.

OR

A TV tower is erected on the ground as shown in the figure below. Two ends of the tower are XY and XZ, and BC is the support to keep the two ends from falling apart. If BC is parallel to the ground, then find the distance YZ.



30. If $\cos\theta = \dfrac{7}{25}$, find the values of all T-ratios of θ. [3]

31. A box contains 20 balls bearing numbers 1, 2, 3,..., 20, respectively. A ball is taken out at random from the box. Find the probability that the number on the ball is [3]

i. an odd number
ii. divisible by 2 or 3
iii. a prime number
iv. not divisible by 10

Section D

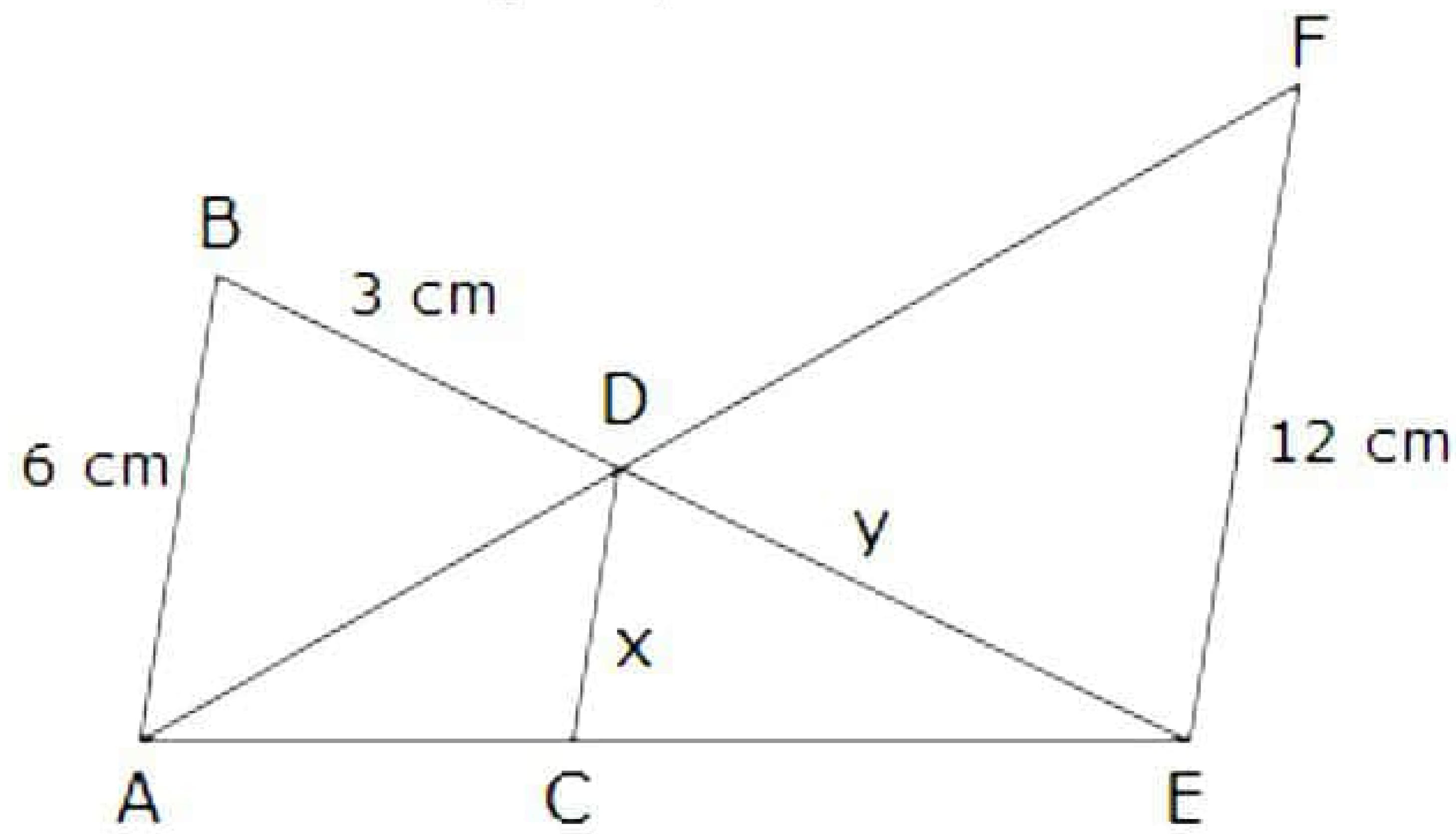
Section D consists of 4 questions of 5 marks each.

32. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. If one pipe takes 3 minutes more than the other to fill it, then find the time in which each pipe would fill the cistern. [5]

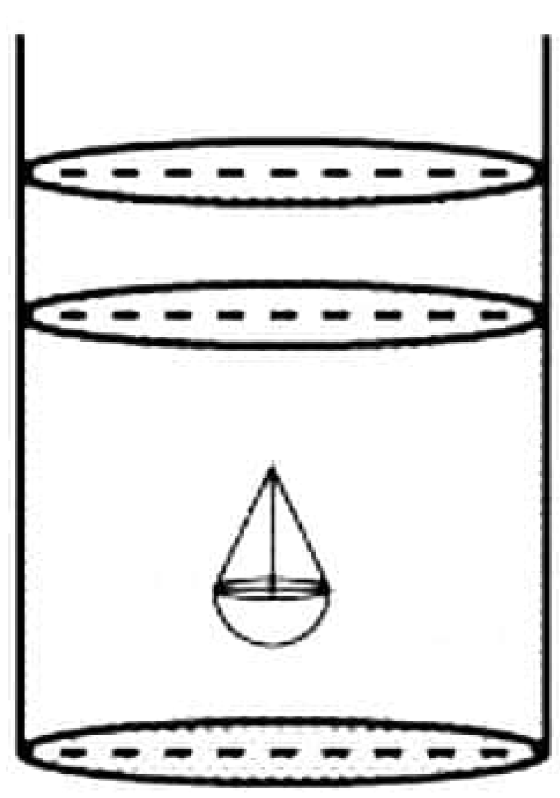
OR

A two-digit number is such that the product of its digits is 48. When 18 is subtracted from the number, the digits interchange their places. Find the number.

33. In the below figure, AB ∥ CD ∥ EF. Find the values of x and y.



34. A solid is in the shape of a hemisphere of radius 7 cm, surmounted by a cone of height 4 cm. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is 14 cm, find the rise in the water level. [5]



OR

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 400 per m^2.

35. 100 surnames were randomly picked up from a local telephone directory, and the distribution of the number of letters of the English alphabet in the surnames was obtained as follows: [5]

No. of letters	No. of surnames
1–4	6
4–7	30
7–10	40
10–13	16
13–16	4
16–19	4

Determine the median and mean number of letters in the surname. Also, find the modal size of surnames.

Section E

Case study based questions are compulsory.

36. Rukhsar is celebrating her birthday. She invited her friends. She bought a packet of chocolates which contains 120 chocolates. She arranges the chocolates such that in the first row there are 3 chocolates, 4 chocolates in second row, 7 chocolates in third row and so on.

i. Find the total number of rows of chocolates. [2]

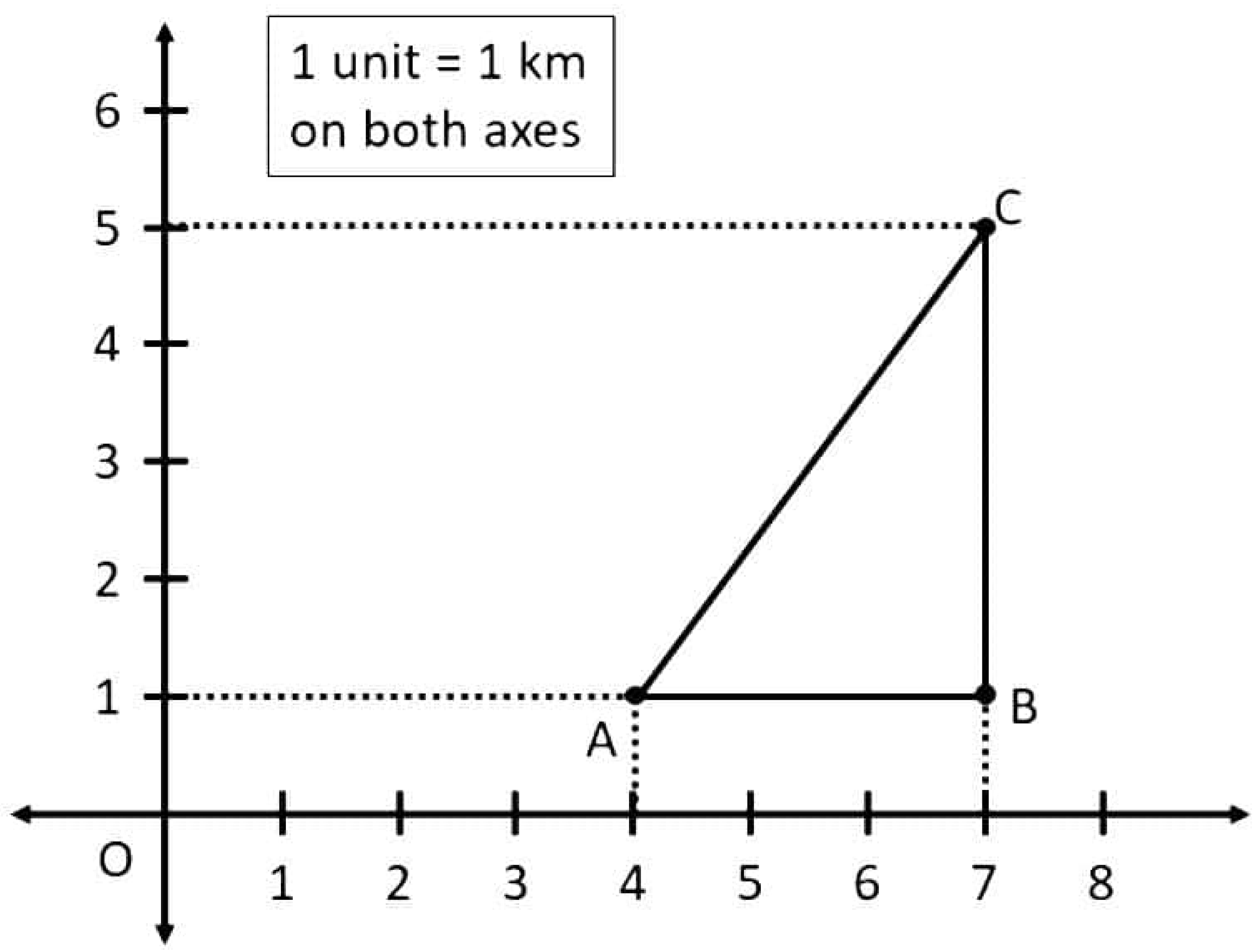
OR

If Rukhsar decides to make 15 rows, then how many total chocolates will be placed by her with the same arrangement? [2]

ii. How many chocolates are placed in last row? [1]

iii. Find the difference in number of chocolates placed in 7^{th} and 3^{rd} row. [1]

37. The location of homes of three friends Ajay, Bipin and Chandu are shown by the points A, B and C respectively. Now using the given information, answer the following questions.



i. Find the distance between Ajay's house and Bipin's house. [1]

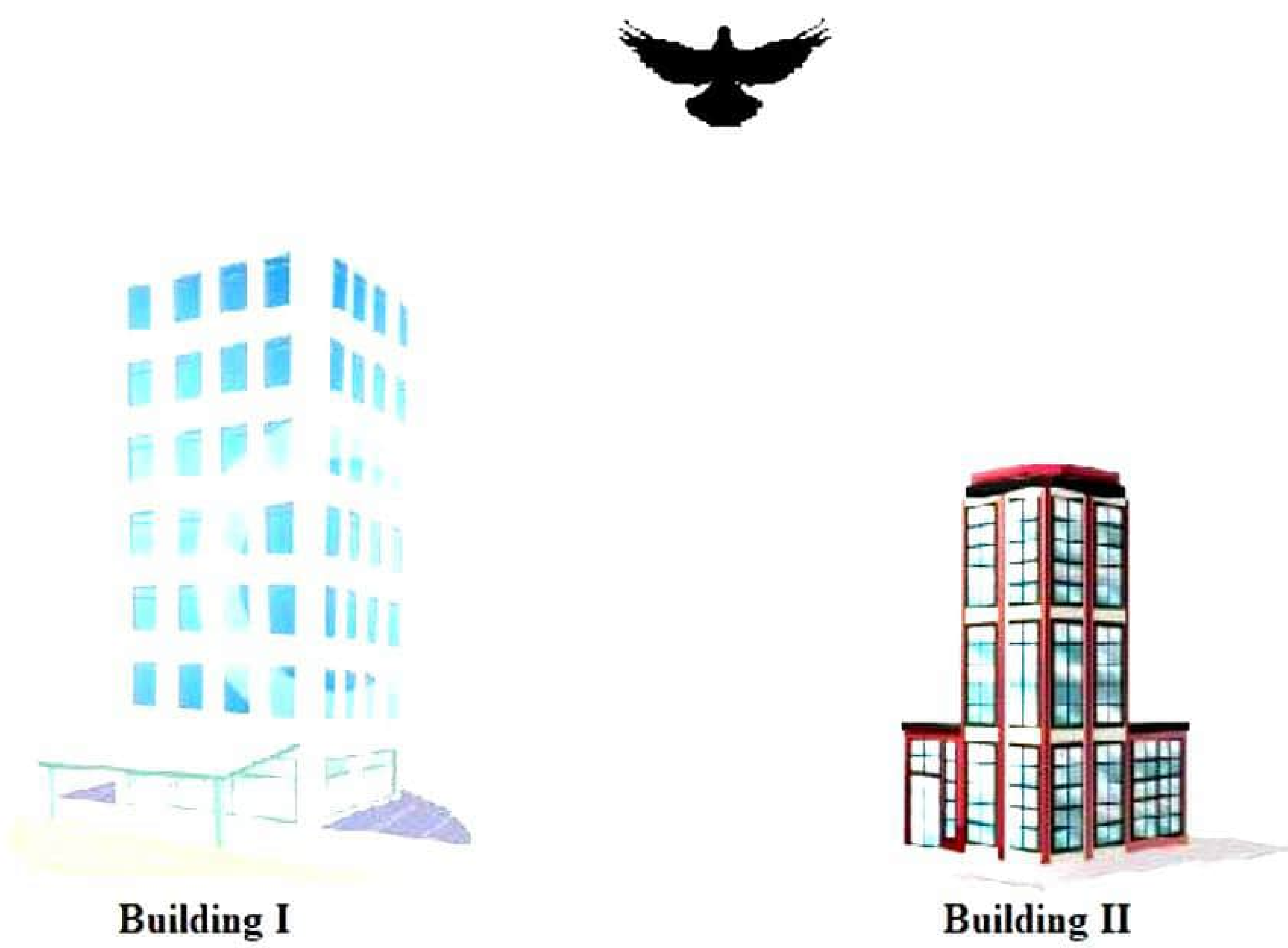
ii. Find the distance between Chandu's house and Bipin's house. [1]

iii. Find the distance between Chandu's house and Ajay's house. [2]

OR

Find the difference between the longest and the shortest route from Ajay's to Chandu's house. [2]

38. A bird flying at height h can see the top of the two buildings of height 534 m and 300 m. The angles of depression from bird, to the top of first and second buildings are 30° and 60° respectively. If the distance between the two buildings is 142 m, and the bird is vertically above the midpoint of the distance between the two buildings, answer the following questions.



i. The distance between bird and top of building I is [2]

OR

The distance between bird and top of building II is [2]

ii. Find the approximate height at which the bird is flying. [1]

iii. Find the angle of elevation if the bird is at the ground and its distance from building II is 300 m. [1]

Solution

Section A

1. Correct option: B

Explanation:

12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$LCM = 2^2 \times 3 \times 5 \times 7 = 420$

2. Correct option: A

Explanation:

$x^2 - 3x - 10$

$= x^2 - 5x + 2x - 10$

$= x(x-5) + 2(x-5)$

$= (x-5)(x+2)$

$\therefore \ (x-5)(x+2) = 0$

i.e., x = 5 or x = −2

3. Correct option: C

Explanation:

Let the required polynomial be $ax^2 + bx + c$, and let its zeroes be α and β.

$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$

$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

If $a = 4k$, then $b = -k$, $c = -4k$

Therefore, the quadratic polynomial is $k(4x^2 - x - 4) = 0$, where k is a real number .

4. Correct option: B

Explanation:

5x − 4y + 8 = 0

7x + 6y − 9 = 0

Comparing these equations with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 5,\ b_1 = -4,\ \ c_1 = 8$

$a_2 = 7,\ b_2 = 6,\ \ \ c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}$

$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the given pair of equations intersect at exactly one point.

5. Correct option: D
Explanation:
If p, s, q are in A.P.,
s = p + d   [d = common difference]
q = p + 2d
So, p + q = 2p + 2d = 2(p + d) = 2s

6. Correct option: C
Explanation:
If a point lies in the 3rd quadrant, then its x-coordinate as well as its y-coordinate will be negative.

7. Correct option: B
Explanation:

Distance between two points (x_1, y_1) and (x_2, y_2) $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Then, distance between points $(0,0)$ and $(36,15)$

$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$

$= \sqrt{1296 + 225} = \sqrt{1521} = 39$

8. Correct option: D
Explanation:
A circle can have infinite tangents.

9. Correct option: D
Explanation:
SSA is not a test of similarity, the angle should be included between the two sides.

10. Correct option: D
Explanation:
Corresponding angles of similar triangles are equal.

11. Correct option: D

Explanation:

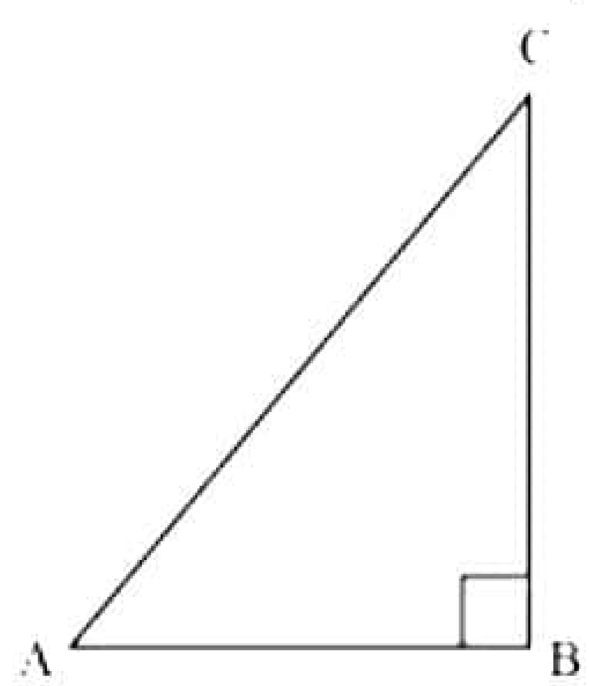
$$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$$\Rightarrow \tan^2\theta = \frac{64}{49}$$

12. Correct option: C

Explanation:

Let ΔABC be a right-angled triangle, right angled at point B.



Given that

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3K.

So AC will be 4K where K is a positive integer.

Now applying Pythagoras theorem in ΔABC,

$AC^2 = AB^2 + BC^2$

$(4K)^2 = AB^2 + (3K)^2$

$16K^2 - 9K^2 = AB^2$

$7K^2 = AB^2$

$AB = \sqrt{7}K$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{side adjacent to } \angle A}$$

$$= \frac{BC}{AB} = \frac{3K}{\sqrt{7}K} = \frac{3}{\sqrt{7}}$$

13. Correct option: D

Explanation:

Given,

No. of errors = 0

Also, No. of errors = $\cos\theta$

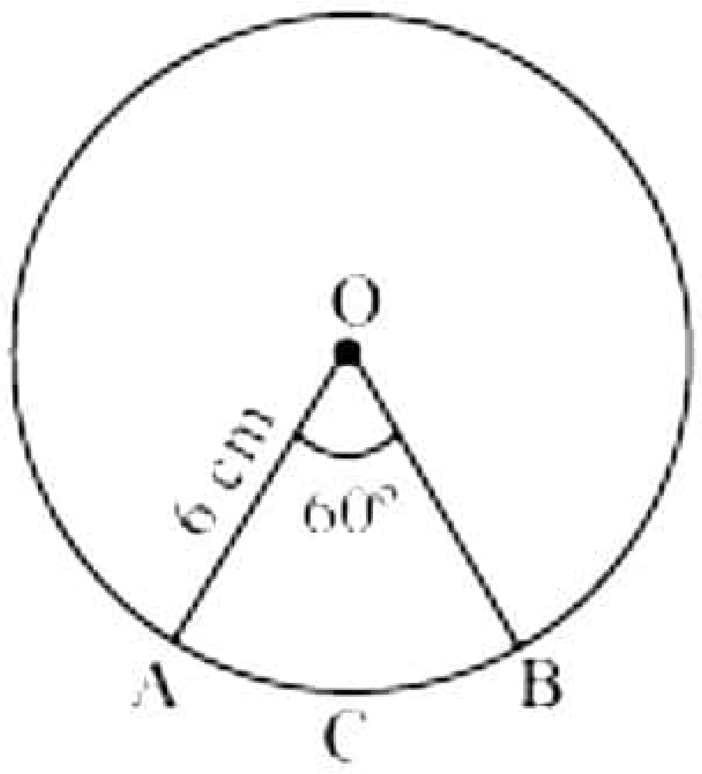
Hence, $\cos\theta = 0$

But $\cos 90^\circ = 0$

Therefore, $\theta = 90^\circ$.

14. Correct option: C

Explanation:



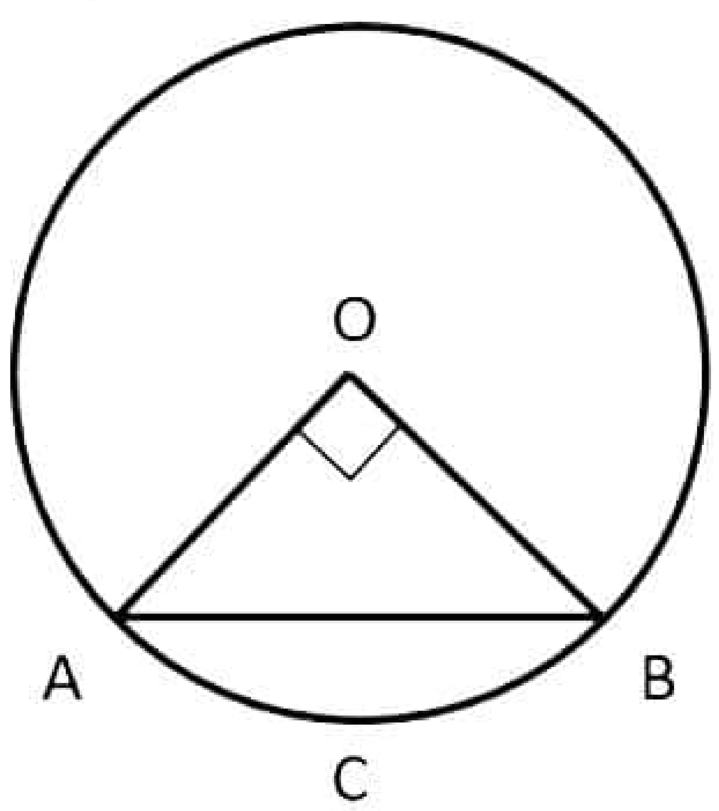
Let OACB be a sector of circle making 60° angle at the centre O of circle.

Area of sector with angle $\theta = \frac{\theta}{360°} \times \pi r^2$

So, area of sector OACB $= \frac{60°}{360°} \times \frac{22}{7} \times (6)^2 = \frac{1}{6} \times \frac{22}{7} \times 6 \times 6 = \frac{132}{7}$ cm^2

15. Correct option: A

Explanation:



Let AB be the chord of a circle subtending 90° angle at the centre O of circle.

Area of minor sector OACB $= \frac{90°}{360°} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 10 \times 10 = \frac{1100}{14} = 78.6$ cm^2

Area of ΔOAB $= \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50$ cm^2

Area of minor segment ACB = Area of minor sector OACB – Area of ΔOAB

= 78.6 – 50 = 28.6 cm^2

16. Correct option: D

Explanation:

For a group of observations, the middle-most value is the median.

So, to find the middle-most age, we must use the formula of median.

And, median $= l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$

17. Correct option: C

Explanation:

When two coins are tossed simultaneously, all possible outcomes are HH, HT, TH, TT.

Total number of possible outcomes = 4

Let E be the event of getting at the most one head.

So, the favourable outcomes are HT, TH, TT.

Number of favourable outcomes = 3

$\therefore$ P(getting at the most 1 head) = P(E) = $\frac{3}{4}$

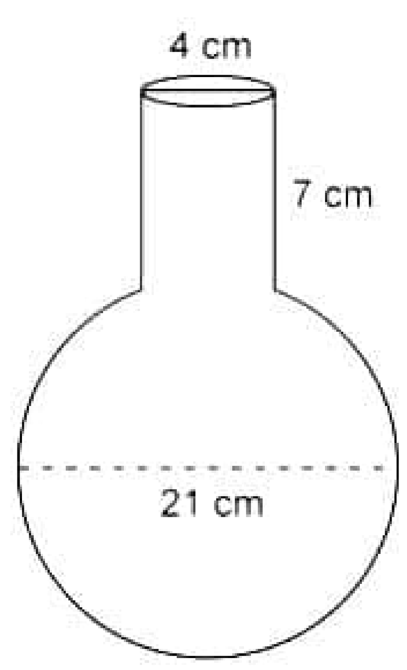
18. Correct option: B

Explanation:

Mode is the observation with the highest frequency, which is 120.

19. Correct option: A

Explanation:



Diameter of the spherical part of vessel = 21 cm

Its radius $= \frac{21}{2}$ cm

Its volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 11 \times 21 \times 21 \text{ cm}^3 = 4851 \text{ cm}^3$$

Volume of cylindrical part of vessel

$$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 7 \text{ cm}^3$$

$$= 88 \text{ cm}^3$$

Now, Quantity of water it can hold = volume of spherical glass vessel + volume of cylindrical neck

Hence, reason (R) is true.

$\therefore$ Quantity of water it can hold = 4851 + 88 = 4939 cm^3.

Thus, assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A).

20. Correct option: A

Explanation:

The statement given in reason is correct and hence, reason is true.

Given system of equations is $x - y = 4$ and $x + y = 6$.

Substituting $x = p$ and $y = 2q$,

$p - 2q = 4$ ....(i)

$p + 2q = 6$ ....(ii)

Adding (i) and (ii),

$2p = 10 \Rightarrow p = 5$

Hence, assertion is true.

Section B

21. Given equations are as follows:

$x - 2y = 6$ ....(1)

$2x + 2y = 6$ ....(2)

Substituting $x = 4$ and $y = -1$ in equations (1) and (2), we have

L.H.S. $= 4 - 2(-1) = 4 + 2 = 6 =$ R.H.S.

L.H.S. $= 2(4) + 2(-1) = 8 - 2 = 6 =$ R.H.S.

Since $x = 4$ and $y = -1$ satisfy both the equations, $(4, -1)$ is a solution of given equations.

22. In ΔLMN and ΔPQR,

$$\frac{PQ}{LM} = \frac{2}{4} = \frac{1}{2}, \quad \frac{QR}{MN} = \frac{3}{9} = \frac{1}{3}$$

$$\because \frac{1}{2} \neq \frac{1}{3} \Rightarrow \frac{PQ}{LM} \neq \frac{QR}{MN}$$

The corresponding sides of ΔLMN and ΔPQR are not proportional.

Hence, ΔLMN is not similar to ΔPQR.

23. Two tangents PA and PB are drawn from an external point P to a circle with centre O.

Since the tangents from an external point to a circle are equal,

PA = PB ....(i)

Also, CA = CE and DB = DE

Perimeter of ΔPCD = PC + CD + PD

= (PA − CA) + (CE + DE) + (PB − DB)

= (PA − CE) + (CE + DE) + (PB − DE)

= PA + PB

= 2PA

= 2 × 14 cm

= 28 cm

Hence, the perimeter of ΔPCD is 28 cm.

24. Area of a sector $= \frac{\theta}{360^\circ} \times \pi r^2$ ....(1)

Length of an arc, $l = \frac{\theta}{360^\circ} \times 2\pi r$

Then, $\frac{\theta}{360^\circ} = \frac{l}{2\pi r}$ ....(2)

Substituting (2) in (1),

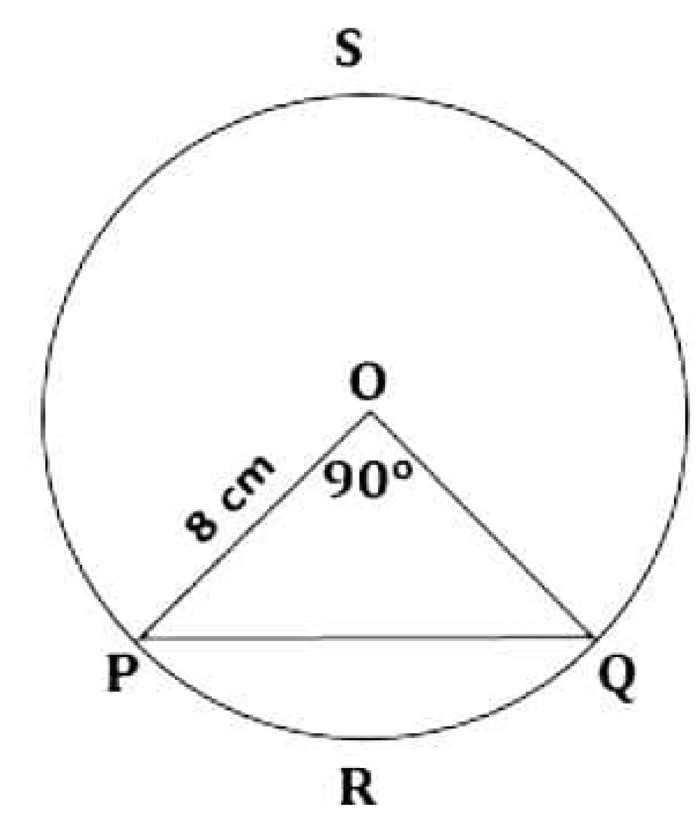
Area of a sector $= \frac{l}{2\pi r} \times \pi r^2$

$= \frac{1}{2} \times l \times r$

= Half the product of length of an arc and radius

OR

Let PQ be the chord of a circle subtending 90° angle at centre O.



(i) Area of minor sector OPRQ $= \frac{90^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 8 \times 8$

$= \frac{352}{7}$ cm^2

= 50.29 cm^2

Area of $\Delta POQ = \frac{1}{2} \times OP \times OQ$

$= \frac{1}{2} \times 8 \times 8$

= 32 cm^2

Area of minor segment PRQ = Area of minor sector OPRQ – Area of ΔPOQ

= (50.29 – 32) cm^2

= 18.29 cm^2

(ii) Area of major sector OPSQ $= \left(\frac{360^\circ - 90^\circ}{360^\circ}\right) \times \pi r^2$

$= \left(\frac{270^\circ}{360^\circ}\right) \pi r^2$

$= \frac{3}{4} \times \frac{22}{7} \times 8 \times 8$

= 150.86 cm^2

25.

$$\text{L.H.S.} = \sqrt{\frac{1-\cos A}{1+\cos A}} + \sqrt{\frac{1+\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}} + \sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} + \sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} + \frac{1+\cos A}{\sin A}$$

$$= \frac{1-\cos A + 1 + \cos A}{\sin A}$$

$$= \frac{2}{\sin A}$$

$$= 2\,\text{cosec}\ A$$

$$= \text{R.H.S.}$$

OR

$$\text{L.H.S.} = (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= (\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)$$

$$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin\theta}{\sin\theta\cos\theta} + \frac{\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$

$$= \sec\theta + \text{cosec}\,\theta$$

$$= \text{R.H.S.}$$

Section C

26. To find the minimum number of rooms required, first find the maximum number of participants which can be accommodated in each room such that the number of participants in each room is the same.

This can be determined by finding the HCF of 60, 84 and 108.

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

$\text{HCF} = 2^2 \times 3 = 12$

So, the minimum number of rooms required $= \dfrac{\text{Total number of participants}}{12}$

$= \dfrac{60 + 84 + 108}{12}$

$= 21$

27. $t^2 - 15 = 0$

$\Rightarrow t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$

$\Rightarrow t = \sqrt{15}$ or $t = -\sqrt{15}$

So, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes $= \sqrt{15} + \left(-\sqrt{15}\right) = 0 = \dfrac{-0}{1} = \dfrac{-(\text{Coefficient of } t)}{\left(\text{Coefficient of } t^2\right)}$

Product of zeroes $= \left(\sqrt{15}\right)\left(-\sqrt{15}\right) = -15 = \dfrac{-15}{1} = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

28. Let the speed of train be x km/h.

Time taken to travel 480 km $= \dfrac{480}{x}$ hrs

In second condition, speed of train $= (x - 8)$ km/h

It is also given that the train will take 3 more hours to cover the same distance.

Therefore, time taken to travel 480 km $= \left(\dfrac{480}{x} + 3\right)$ hrs

Speed × Time = Distance

$$(x - 8)\left(\frac{480}{x} + 3\right) = 480$$

$\Rightarrow 480 + 3x - \dfrac{3840}{x} - 24 = 480$

$\Rightarrow 3x - \dfrac{3840}{x} - 24 = 0$

$\Rightarrow 3x^2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

So, the required quadratic equation is $x^2 - 8x - 1280 = 0$.

OR

Let the age of Jacob be x and the age of his son be y.
According to the given information,

$(x + 5) = 3(y + 5)$

$x - 3y = 10 \quad \dots (1)$

$(x - 5) = 7(y - 5)$

$x - 7y = -30 \quad \dots (2)$

From (1), we obtain
$x = 3y + 10 \quad \dots (3)$
Substituting this value in equation (2), we obtain

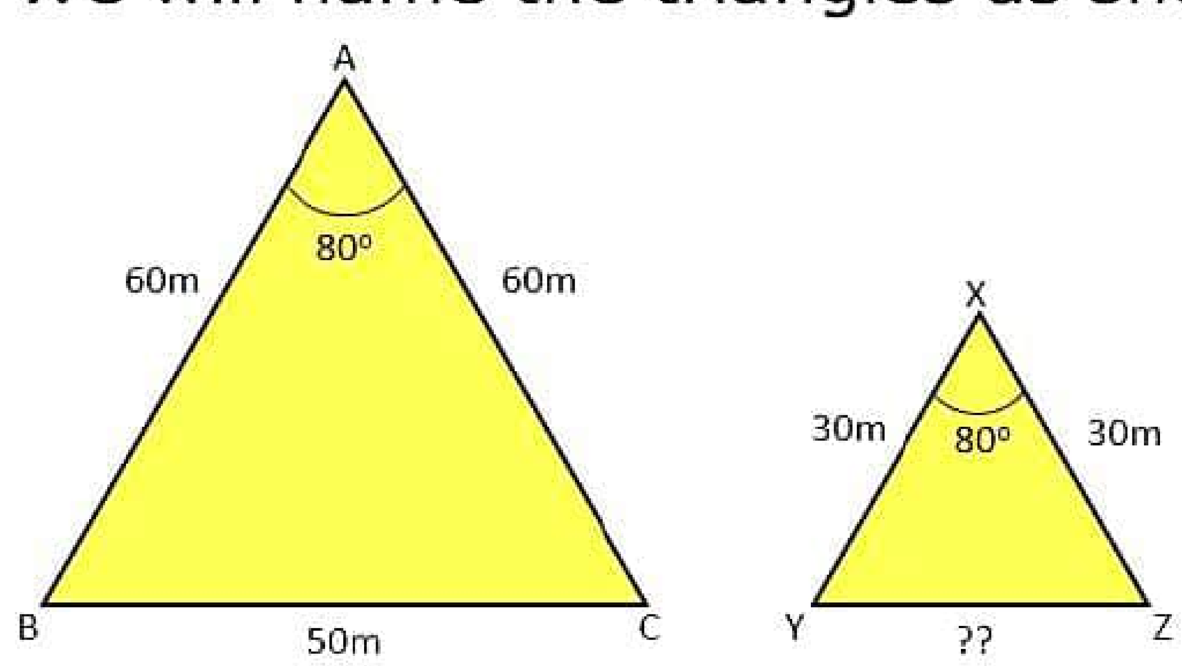
$3y + 10 - 7y = -30$

$-4y = -40$

$y = 10 \quad \dots (4)$

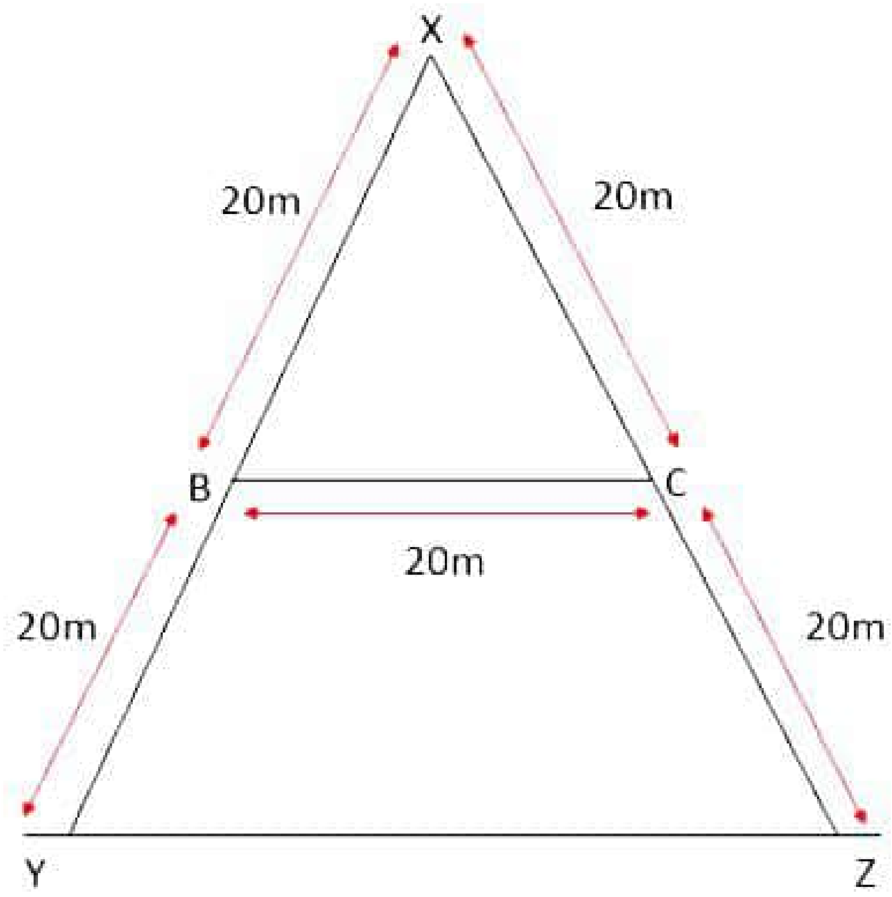
Substituting this value in equation (3), we obtain
$x = 3 \times 10 + 10 = 40$
Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

29. We will name the triangles as shown below:



In △ABC and △XYZ, we have
AB/XY = 2
AC/XZ = 2
Also, ∠A = ∠X
∴ △ABC ~ △XYZ … (SAS test)
⇒ BC/YZ = AB/XY … (Corresponding sides of similar triangles)
⇒ BC/YZ = 2
∴ YZ = ½ BC
∴ YZ = 25 m
Hence, the base of the smaller pyramid is 25 m.

OR



Here in △XBC and △XYZ, we have

XB/XY = 20/40 = ½

XC/XZ = 20/40 = ½

Also, ∠BXC = ∠YXZ ... (common angle)

∴ △XBC ~△XYZ ... (SAS test)

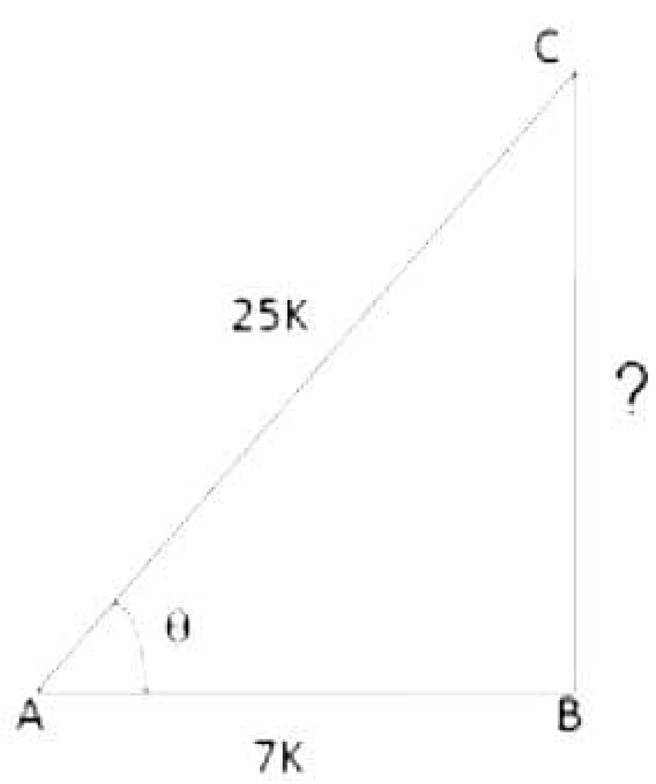
⇒ BC/YZ = XB/XY ... (Corresponding sides of similar triangles)

⇒ BC/YZ =½

∴ YZ = 2(BC) = 2 × 20 = 40 m

Hence, the distance YZ is 40 m.

30.



Given : $\cos\theta = \dfrac{7}{25}$

Let AB = 7k and AC = 25k, where k is positive

Let us draw ΔABC in which ∠B = 90° and ∠BAC = θ.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

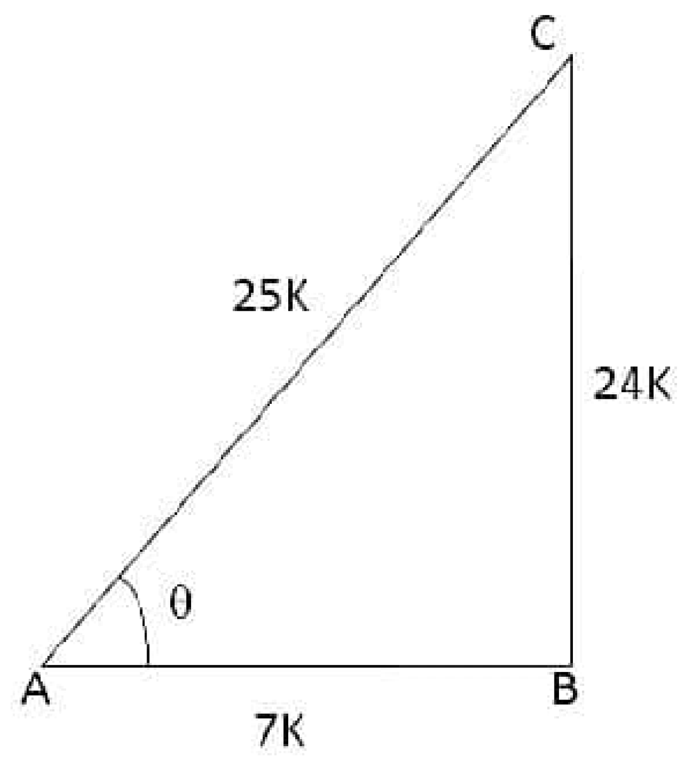
$\Rightarrow BC^2 = AC^2 - AB^2$

$BC^2 = \left[(25k)^2 - (7k)^2\right]$

$= \left(625k^2 - 49k^2\right)$

$= 576k^2$

$\Rightarrow BC = \sqrt{576k^2} = 24k$



$\therefore \sin\theta = \frac{BC}{AC} = \frac{24k}{25k} = \frac{24}{25}; \cos\theta = \frac{7}{25}$ (given)

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{24}{25} \times \frac{25}{7}\right) = \frac{24}{7}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{25}{24}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{25}{7}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{7}{24}$

31. Total number of balls = 20

balls = 20

i. Odd numbers are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.
   Total no. of odd numbers = 10
   $\therefore$P(getting an odd number) = $\frac{10}{20} = \frac{1}{2}$

ii. Numbers divisible by 2 or 3 are 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.
   Total no. of numbers divisible by 2 or 3 = 13
   P(getting a number divisible by 2 or 3) = $\frac{13}{20}$

iii. Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19
   Total no. of prime numbers = 8
   P(getting a prime number) = $\frac{8}{20} = \frac{2}{5}$

iv. Numbers divisible by 10 are 10, 20.
   Total numbers divisible by 10 = 2
   $\therefore$P(getting a number not divisible by 10) = $\left(1 - \frac{2}{20}\right) = \frac{18}{20} = \frac{9}{10}$

Section D

32. Let the faster pipe takes x minutes to fill the cistern.
Then the other pipe takes (x + 3) minutes to fill the cistern.

Total time taken by two pipes = $3\frac{1}{13}$ minutes = $\frac{40}{13}$ minutes

$$\frac{1}{x}+\frac{1}{(x+3)}=\frac{13}{40}$$

$$\Rightarrow \frac{(x+3)+x}{x(x+3)}=\frac{13}{40}$$

$$\Rightarrow 40(2x+3)=13(x^2+3x)$$

$$\Rightarrow 80x+120=13x^2+39x$$

$$\Rightarrow 13x^2-41x-120=0$$

$$\Rightarrow 13x^2-65x+24x-120=0$$

$$\Rightarrow 13x(x-5)+24(x-5)=0$$

$$\Rightarrow (x-5)(13x+24)=0$$

$$\Rightarrow x=5 \quad \text{or} \quad x=\frac{-24}{13}$$

$\Rightarrow x=5$ ($\because$ Time cannot be negative)

If the faster pipe takes 5 minutes to fill the cistern, then the other pipe takes (5 + 3) minutes = 8 minutes to fill the cistern.

OR

Let the tens digit = x

Then, units digit = $\frac{48}{x}$

Then, number = $10x + \frac{48}{x}$

And, number obtained by interchanging the digits = $10\left(\frac{48}{x}\right) + x$

Thus, we have

$$\left(10x + \frac{48}{x}\right) - \left[10\left(\frac{48}{x}\right) + x\right] = 18$$

$$\Rightarrow 10x + \frac{48}{x} - 10\left(\frac{48}{x}\right) - x = 18$$

$$\Rightarrow 9x + \frac{48}{x} - \frac{480}{x} = 18$$

$$\Rightarrow 3x + \frac{16}{x} - \frac{160}{x} = 6 \quad \text{(Dividing by 3 throughout)}$$

$\Rightarrow 3x^2 + 16 - 160 = 6x$

$\Rightarrow 3x^2 - 6x - 144 = 0$

$\Rightarrow x^2 - 2x - 48 = 0$

$\Rightarrow x^2 - 8x + 6x - 48 = 0$

$\Rightarrow x(x - 8) + 6(x - 8) = 0$

$\Rightarrow (x - 8)(x + 6) = 0$

$\Rightarrow (x - 8) = 0$ or $(x + 6) = 0$

$\Rightarrow x = 8$ or $x = -6$

But digit cannot be negative.

$\therefore x = 8$ = tens digit

And, units digit = $\frac{48}{x} = \frac{48}{8} = 6$

Hence, the required two-digit number is 86.

33. In ΔADB and ΔFDE,

$\angle ABD = \angle FED$ (alternate angles)

$\angle BAD = \angle EFD$ (alternate angles)

Therefore by AA criterion of similarity, $\Delta ADB \sim \Delta FDE$.

$$\Rightarrow \frac{AB}{FE} = \frac{BD}{DE}$$

$$\Rightarrow \frac{6}{12} = \frac{3}{y}$$

$$\Rightarrow y = \frac{3 \times 12}{6} = 6 \text{ cm}$$

In ΔABE and ΔCDE,

$\angle ABE = \angle CDE$ (corresponding angles)

$\angle BAE = \angle DCE$ (corresponding angles)

Therefore by AA criterion of similarity, $\Delta ABE \sim \Delta CDE$.

$\Rightarrow \frac{AB}{CD} = \frac{BE}{DE}$

$\Rightarrow \frac{6}{x} = \frac{3+6}{6}$

$\Rightarrow \frac{6}{x} = \frac{9}{6}$

$\Rightarrow x = \frac{6 \times 6}{9} = 4 \text{ cm}$

34. Hemisphere: Radius (r) = 7 cm

Cone: Radius (r) = 7 cm and height (h) = 4 cm

Cylindrical container: Radius (R) = 14 cm

Let the rise in height be H cm.

Volume of the hemisphere = $V_H = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (7 \text{ cm})^3 = \frac{2156}{3} \text{cm}^3$

Volume of cone = $V_C = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (7 \text{ cm})^2 \times 4 \text{ cm} = \frac{616}{3} \text{cm}^3$

Volume of solid is: $V_S = V_H + V_C$

$= \left(\frac{2156}{3} + \frac{616}{3}\right) \text{cm}^3$

$= \left(\frac{2772}{3}\right) \text{cm}^3$

$= 924 \text{ cm}^3$

Now, volume of water displaced is equal to the volume of solid as the water is in the cylindrical container.

⇒ Volume of solid = Volume of cylinder with radius 'R' and height 'H'

$\Rightarrow 924 = \pi R^2 H$

$\Rightarrow 924 = \frac{22}{7} \times 14 \times 14 \times H$

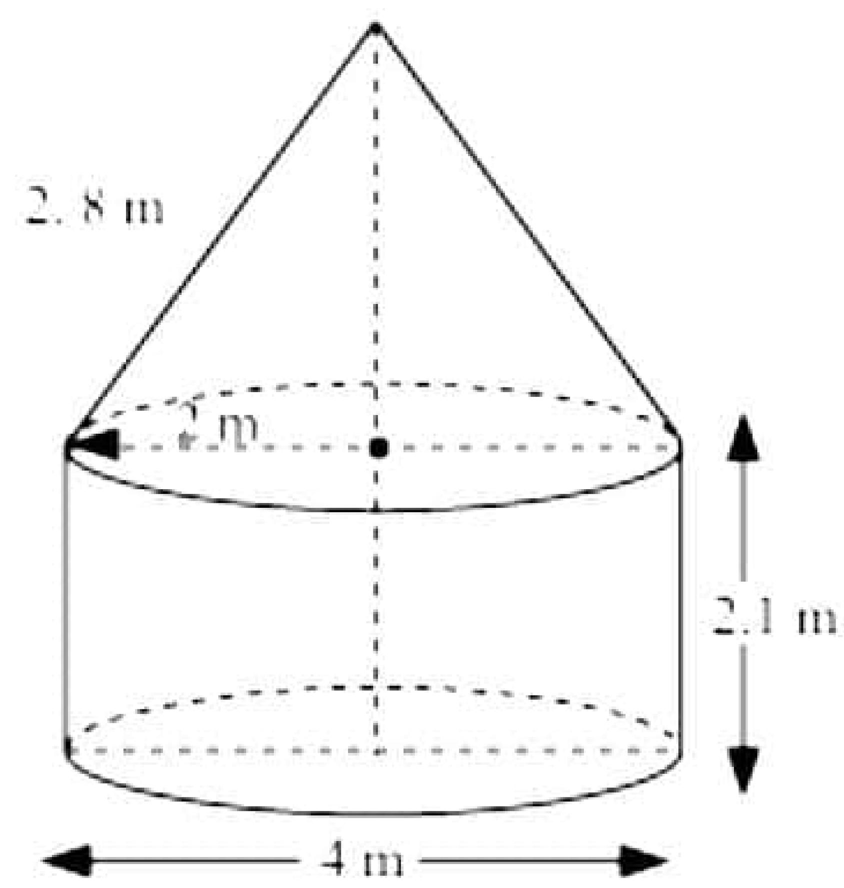
$\Rightarrow 924 = 22 \times 2 \times 14 \times H$

$\Rightarrow H = \frac{924}{22 \times 2 \times 14}$

$\Rightarrow H = 1.5 \text{ cm}$

Thus, the rise in the water level is by 1.5 cm.

OR



Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

So, radius (r) of the cylindrical part = 2 m

Slant height (l) of a conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi(2.8 + 4.2)$

$= 2\pi \times 7\ m^2$

$= 2 \times \frac{22}{7} \times 7$

$= 44\ m^2$

Now, cost of 1 m^2 canvas = Rs. 400

Then, cost of 44 m^2 canvas = 44 × 400 = Rs. 17,600

So, it will cost Rs. 17,600 for making a tent.

35. Let the assumed mean A be 8.5. Class interval h = 3.

Class	Frequency f_i	Mid-value x_i	$u_i = \left(\frac{x_i - A}{h}\right)$	$f_i u_i$	C.F.
1-4	6	2.5	-2	-12	6
4-7	30	5.5	-1	-30	36
7-10	40	8.5 = A	0	0	76
10-13	16	11.5	1	16	92
13-16	4	14.5	2	8	96
16-19	4	17.5	3	12	100
	N = 100				

N = Total frequency = 100

i. Mean $= A + h\left(\frac{\sum f_i u_i}{N}\right)$

$= 8.5 - 3 \times \left(\frac{-6}{100}\right)$

$= 8.5 - \frac{18}{100}$

$= 8.5 - 0.18$

$= 8.32$

ii. $\frac{N}{2} = 50$; Cumulative frequency just after 50 is 76.

∴ Median class is 7–10.

∴ l = 7, h = 3, N = 100, f = 40, cf = 36

Median $= l + h\left(\frac{\frac{N}{2} - cf}{f}\right)$

$= 7 + 3 \times \left(\frac{50 - 36}{40}\right)$

$= 7 + \frac{21}{20}$

$= 7 + 1.05$

$= 8.05$

iii. Mode = 3 × Median – 2 × Mean

= 3 × 8.05 – 2 × 8.32

= 24.15 – 16.64

= 7.51

Thus, mean = 8.32, median = 8.05 and mode = 7.51.

Section E

36.

i. Here, the chocolates are arranged in increasing order of 2.

Thus, it forms an A.P. with a = 3 and d = 2.

Therefore, the required A.P. is 3, 5, 7, ...

Given, $S_n = 120$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 120 = \frac{n}{2}\left[2\times 3 + (n-1)2\right]$$

$$\Rightarrow 240 = \left(6n + 2n^2 - 2n\right)$$

$$\Rightarrow n^2 + 2n - 120 = 0$$

$$\Rightarrow (n+12)(n-10) = 0$$

$$\Rightarrow (n+12) = 0 \text{ or } (n-10) = 0$$

$$\Rightarrow n = -12 \text{ or } n = 10$$

Number of rows can't be negative.

Hence, total number of rows of chocolates is 10.

OR

Here, n = 15

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{15} = \frac{15}{2}\left[2\times 3 + 14\times 2\right] = \frac{15\times 34}{2} = 255$$

Hence, 255 chocolates will be placed by her with the same arrangement.

ii. Here, a = 3, d = 2 and n = 10

$a_n = a_{10} = a + (n - 1)d = 3 + (10 - 1)2 = 21$

Hence, 21 chocolates are placed in last row.

iii. We have, d = 2 and $a_n = a + (n - 1)d$

$\Rightarrow a_7 - a_3 = a + 6d - a - 2d = 4d = 4(2) = 8$

Hence, the difference in number of chocolates placed in 7th and 3rd row is 8.

37.

i.

$A(4,1)$ and $B(7,1)$

$d(AB) = \sqrt{(7-4)^2 + (1-1)^2} = 3$ km

ii.

$C(7,5)$ and $B(7,1)$

$d(BC) = \sqrt{(7-7)^2 + (5-1)^2} = 4$ km

iii.

$C(7,5)$ and $A(4,1)$

$d(CA) = \sqrt{(7-4)^2 + (5-1)^2} = 5$ km

OR

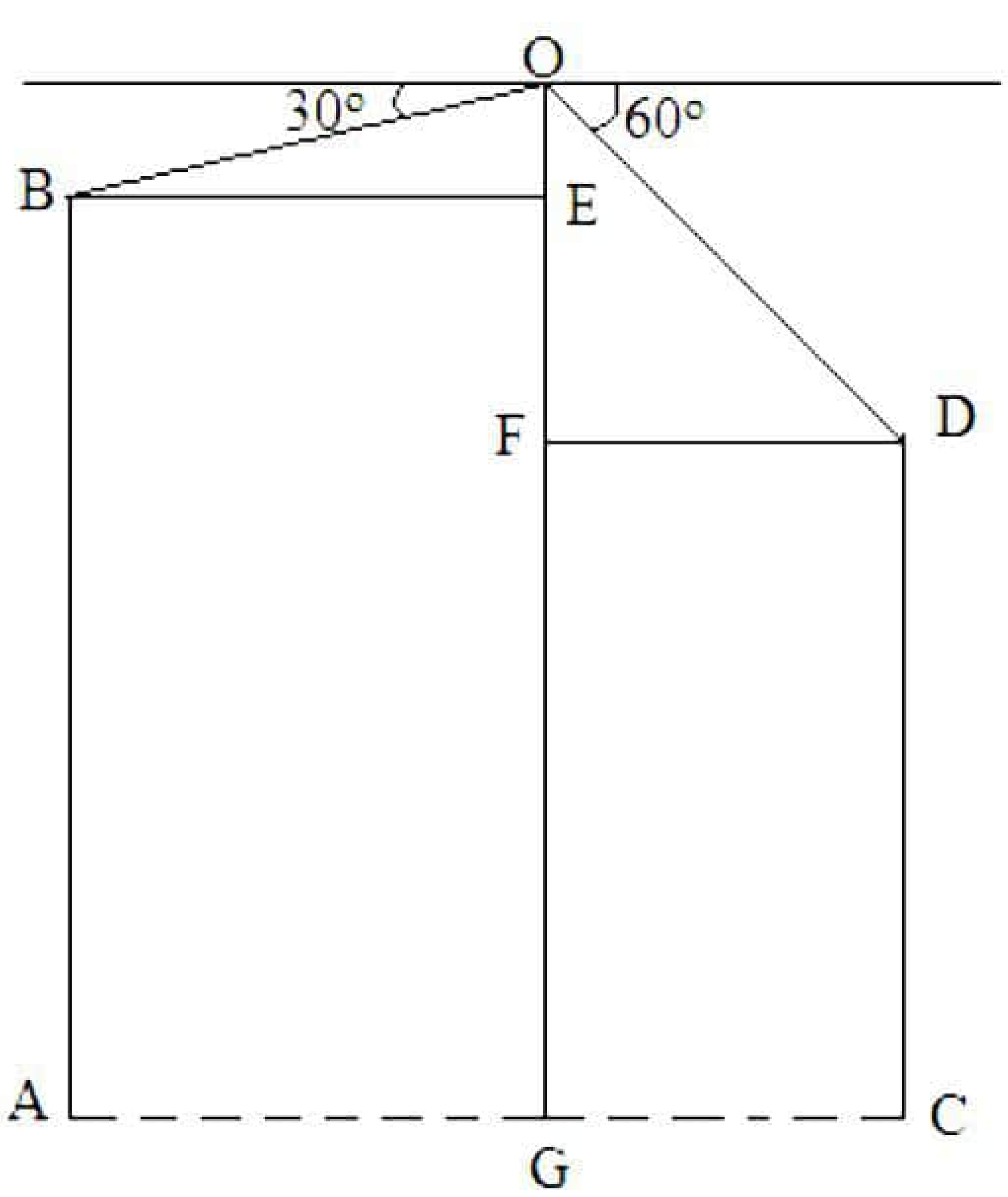
Longest route (via Bipin's home) → A-B-C = 7 km
Shortest route → A-C
$C(7,5)$ and $A(4,1)$

$d(CA) = \sqrt{(7-4)^2 + (5-1)^2} = 5$ km

Difference = 2 km.

38.

i. Let AB and CD represent buildings I and II respectively.



$$BE = \frac{AC}{2} = \frac{142}{2} = 71 \text{ m}$$

$$\cos(\angle OBE) = \frac{BE}{BO}$$

$$\Rightarrow \cos 30^\circ = \frac{71}{BO}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{71}{BO}$$

$$\Rightarrow BO = \frac{142}{\sqrt{3}} \text{m}$$

OR

$$FD = \frac{AC}{2} = \frac{142}{2} = 71 \text{ m}$$

$$\cos(\angle ODF) = \frac{FD}{OD}$$

$$\Rightarrow \cos 60^\circ = \frac{71}{OD}$$

$$\Rightarrow \frac{1}{2} = \frac{71}{OD}$$

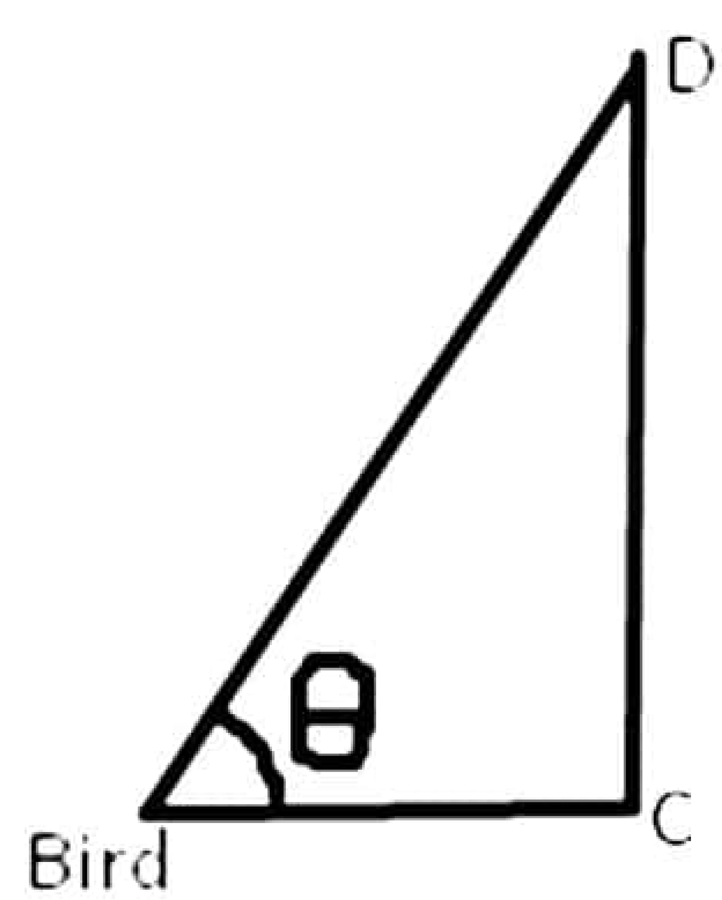
$$\Rightarrow OD = 142 \text{ m}$$

ii. Distance between bird and ground = OG = OE + EG

$$OE = BE \times \tan(\angle OBE) = 71 \times \frac{1}{\sqrt{3}} = \frac{71}{\sqrt{3}} = 40.99 \text{ m}$$

Therefore, OG = 534 + 40.99 = 574.99 m

iii.



tan θ = 300/300 = 1

Therefore, θ = 45°