Class X Session 2024-25
Subject - Mathematics (Standard)
Sample Question Paper - 8

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. A bag contains 8 red, 2 black and 5 white balls. One ball is drawn at random. What is the probability that the ball drawn is not black? **[1]**

a) $\frac{13}{15}$ b) $\frac{1}{3}$

c) $\frac{8}{15}$ d) $\frac{2}{15}$

2. If $ax^2 + bx + c = 0$ has equal roots, then c is equal to **[1]**

a) $\frac{b^2}{2a}$ b) $\frac{b^2}{4a}$

c) $\frac{-b^2}{4a}$ d) $-\frac{b^2}{2a}$

3. A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is r, then the volume of the cylinder is **[1]**

a) $2\pi r^3$ b) $8\pi r^3$

c) $\frac{8}{3}\pi r^3$ d) $4\pi r^3$

4. Let b = a + c. Then the equation $ax^2 + bx + c = 0$ has equal roots if **[1]**

a) a = -c b) a = c

c) a = -2c d) a = 2c

5. Find the sum of the progression: (5 + 13 + 21 + ... + 181) **[1]**

a) 2139 b) 2337

c) 2219 d) 2476

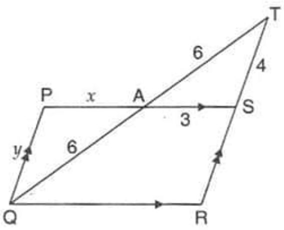
6. A circle drawn with origin as the centre passes through $(\frac{13}{2}, 0)$. The point which does not lie in the interior of the circle is **[1]**

a) $\frac{-3}{4}, 1$
b) $2, \frac{7}{3}$
c) $5, \frac{-1}{2}$
d) $\left(-6, \frac{5}{2}\right)$

7. The zeroes of the polynomial $p(x) = x^2 + 4x + 3$ are given by: **[1]**

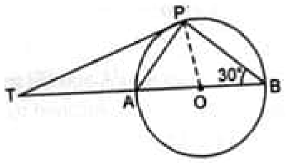
a) -1, 3
b) 1, -3
c) 1, 3
d) -1, -3

8. In the given figure if PS||QR and PQ||SR and AT = AQ = 6, AS = 3, TS = 4, then **[1]**



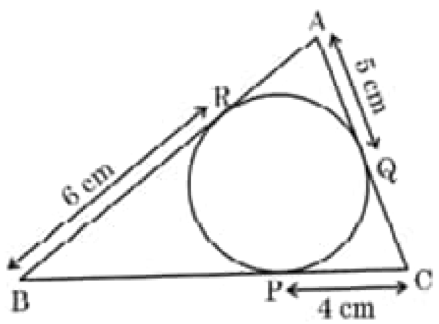
a) x = 2, y = 3.
b) x = 1, y = 2.
c) x = 3, y = 4.
d) x = 4, y = 5.

9. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA extended at T. If $\angle$PBO = 30°, then $\angle$PTA is equals to: **[1]**



a) 20°
b) 30°
c) 40°
d) 60°

10. In the given figure, the perimeter of $\triangle$ABC is: **[1]**



a) 15 cm
b) 30 cm
c) 60 cm
d) 45 cm

11. If $\sec\theta + \tan\theta = p$, then the value of $\sin\theta$ is **[1]**

a) $\frac{1-p^2}{p^2+1}$
b) $\frac{p^2-1}{p^2+1}$
c) $\frac{1+p^2}{p^2-1}$
d) $\frac{p^2+1}{p^2-1}$

12. The ratio of HCF to LCM of the least composite number and the least prime number is: **[1]**

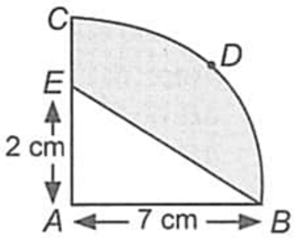
a) 1 : 1
b) 2 : 1

c) 1 : 2 | d) 1 : 3

13. The tops of two poles of height 16 m and 10 m are connected by a wire. If the wire makes an angle of 30^o with the horizontal, then the length of the wire is **[1]**

a) 12 m | b) $10\sqrt{3}$ m
c) 16 m | d) 10 m

14. In the figure, ABDCA represents a quadrant of a circle of radius 7 cm a with centre A. Find the area of the shaded portion. **[1]**



a) 14 cm^2 | b) 31.5 cm^2
c) 24.5 cm^2 | d) 38.5 cm^2

15. A chord of a circle of radius 10 cm subtends a right angle at the centre. The area of the minor segments (given, π = 3.14) is **[1]**

a) 32.5 cm^2 | b) 34.5 cm^2
c) 30.5 cm^2 | d) 28.5 cm^2

16. What is the probability that a leap year has 52 Mondays? **[1]**

a) $\frac{5}{7}$ | b) $\frac{6}{7}$
c) $\frac{2}{7}$ | d) $\frac{4}{7}$

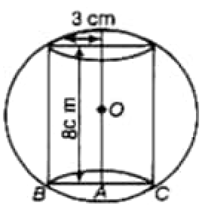
17. Cards marked with numbers 1, 2, 3, ..., 25 are placed in a box and mixed thoroughly and one card is drawn at random from the box. The probability that the number on the card is a multiple of 3 or 5 is **[1]**

a) $\frac{8}{25}$ | b) $\frac{12}{25}$
c) $\frac{4}{25}$ | d) $\frac{1}{5}$

18. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are deviations from a of **[1]**

a) upper limits of the classes | b) lower limits of the classes
c) mid points of the classes | d) frequencies of the class marks

19. **Assertion (A):** In the given figure, a sphere circumscribes a right cylinder whose height is 8 cm and radius of the base is 3 cm. The ratio of the volumes of the sphere and the cylinder is 125 : 54 **[1]**



Reason (R): Ratio of their volume = $\frac{Volume\ of\ sphere}{Volume\ of\ cylinder}$

a) Both A and R are true and R is the correct | b) Both A and R are true but R is not the explanation of A. correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** Three consecutive terms 2k + 1, 3k + 3 and 5k - 1 form an AP than k is equal to 6. **[1]**

Reason (R): In an AP a, a + d, a + 2d, ... the sum to n terms of the AP be $S_n = \frac{n}{2}(2a + (n-1)d)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

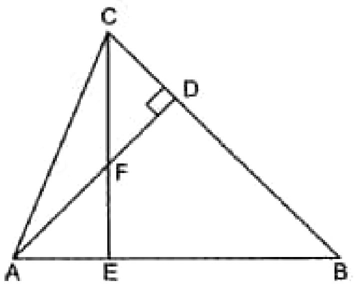
c) A is true but R is false.

d) A is false but R is true.

Section B

21. 2002 cartons of Lassi bottles and 2618 cartons of Frooti are to be stacked in a storeroom. If each stack is of the same height and is to contain cartons of the same type of bottles, what would be the greatest number of cartons each stack would have? **[2]**

22. In Fig. AD and CE are two altitudes of $\triangle$ABC intersect each other at point F. Prove that $\Delta FDC \sim \Delta BEC$ **[2]**



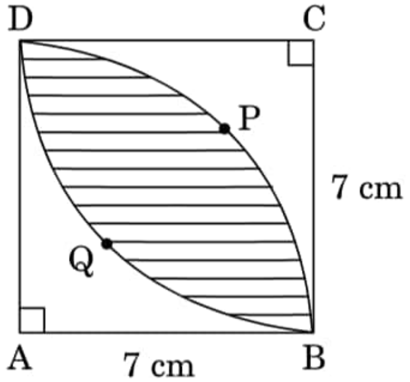
23. Prove that the tangents at the end of a chord of a circle make equal angles with the chord. **[2]**

24. Prove that: $\frac{\cos A}{1-\tan A} + \frac{\sin^2 A}{sinA - cosA}$ = sin A + cos A. **[2]**

OR

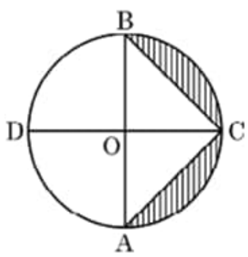
If (3 sin θ + 5 cos θ) = 5, prove that (5 sin θ - 3 cos θ) = $\pm$ 3

25. Calculate the area of the shaded region common between two quadrants of circles of radius 7 cm each (as shown in Figure). **[2]**



OR

In the given figure, AB and CD are the diameters of a circle with centre O, perpendicular to each other. If OA = 7 cm, find the area of the shaded region.



Section C

26. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic wise and the height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and the number of Mathematics books is 336. Assuming that the books are of the same thickness, determine the number of stacks of English, Hindi and Mathematics books. **[3]**

27. A point P divides the line segment joining the points A (3, - 5) and B (- 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of k. **[3]**

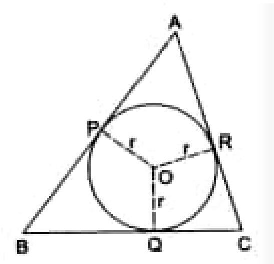
28. Nine times the side of one square exceeds a perimeter of a second square by one metre and six times the area of the second square exceeds twenty-nine times the area of the first by one square metre, Find the side of each square. **[3]**

OR

Solve: $x^2 + 5x - (a^2 + a - 6) = 0$

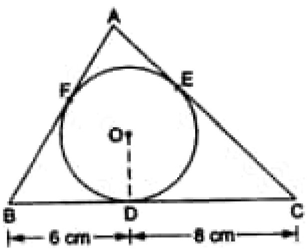
29. In the given figure, the sides AB, BC and CA of a triangle ABC touch a circle with center O and radius r at P, Q and R respectively. Prove that. **[3]**

a. AB + CQ = AC + BQ
b. area $(\Delta ABC) = \frac{1}{2}(\text{perimeter of } \Delta ABC) \times r$.



OR

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8cm respectively. Find the lengths of the sides AB and AC.



30. Prove the identity: **[3]**

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A- cosec^3 A} = \sin^2 A\cos^2 A$$

31. As observed from the top of a light-house, 100 m high above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 60° . Determine the distance travelled by the ship during the period of observation. (Use $\sqrt{3}$ = 1.732) **[3]**

Section D

32. A leading library has a fixed charge for the first three days and an additional charge for each day thereafter Sarika paid ₹ 27 for a book kept for seven days, while Sury paid ₹ 21 for the book she kept for five days, find the fixed charge and the charge for each extra day. **[5]**

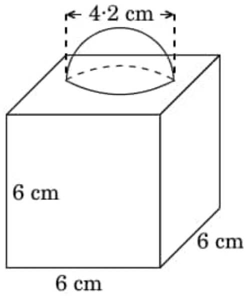
OR

A train covered a certain distance at a uniform speed. If it were 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

33. Points P, Q and R in order are dividing a line segment joining A(1, 6) and B(5, -2) in four equal parts. Find the coordinates of P, Q and R. **[5]**

34. In Figure, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm and the hemisphere fixed on the top has a diameter of 4·2 cm. Find **[5]**

a. the total surface area of the block.
b. the volume of the block formed. (Take $\pi = \frac{22}{7}$)



OR

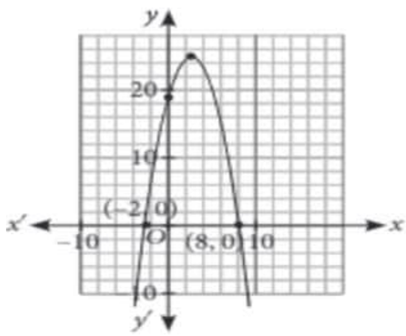
A building is in the form of a cylinder surmounted by a hemispherical dome. The base diameter of the dome is equal to $\frac{2}{3}$ of the total height of the building. Find the height of the building, if it contains $67\frac{1}{21}\text{m}^3$ of air.

35. If the sum of the first p terms of an A.P. is q and the sum of the first q terms is p; then show that the sum of the first (p + q) terms is { -(p + q)}. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Rachna and her husband Amit who is an architect by profession, visited France. They went to see Mont Blanc Tunnel which is a highway tunnel between France and Italy, under the Mont Blanc Mountain in the Alps, and has a parabolic cross-section. The mathematical representation of the tunnel is shown in the graph.



i. What will be the expression of the polynomial given in diagram? (1)
ii. What is the value of the polynomial, represented by the graph, when x = 4? (1)
iii. If the tunnel is represented by $-x^2 + 3x - 2$. Then what is its zeroes? (2)

OR

What is sum of zeros and product of zeros for $-x^2 + 3x - 2$? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Gurpreet is very fond of doing research on plants. She collected some leaves from different plants and measured their lengths in mm.

The data obtained is represented in the following table:

Length (in mm):	70-80	80-90	90-100	100-110	110-120	120-130	130-140
Number of leaves:	3	5	9	12	5	4	2

Based on the above information, answer the following questions:

i. Write the median class of the data.

ii. How many leaves are of length equal to or more than 10 cm?

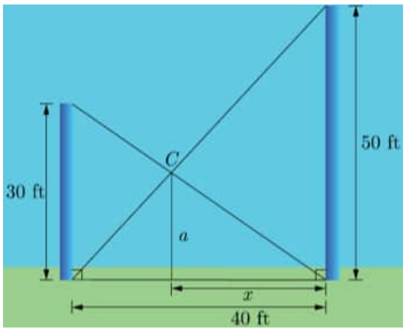
iii. a. Find median of the data.

OR

b. Write the modal class and find the mode of the data.

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



i. What is the horizontal distance from C to the taller pole? (1)

ii. How high above the ground is the coupling? (1)

iii. How far down the wire from the smaller pole is the coupling? (2)

OR

Find the length of line joining the top of the two poles. (2)

Solution

Section A

1. **(a)** $\frac{13}{15}$
   Explanation: Total number of balls in the bag = 8 + 2 + 5 = 15.
   Number of non-black balls = 8 + 5 = 13.
   $\therefore$ P (getting a non-black ball) = $\frac{13}{15}$
2.
   (b) $\frac{b^2}{4a}$
   Explanation: If $ax^2+bx+c=0$ has equal roots, then
   $b^2-4ac=0$
   $\Rightarrow 4ac=b^2$
   $\Rightarrow c=\frac{b^2}{4a}$
3. **(a)** $2\pi r^3$
   Explanation: Volume of a sphere = $(4/3)\pi r^3$
   Volume of a cylinder = $\pi r^2 h$
   Given, sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder and the radius of the sphere is *r*.
   Thus, height of the cylinder = diameter = 2r and base radius = r
   Volume of the cylinder = $\pi \times r^2 \times 2r = 2\pi r^3$
4.
   (b) a = c
   Explanation: Since, If $ax^2 + bx + c = 0$ has equal roots, then
   $b^2 - 4ac = 0$
   $\Rightarrow (a + c)^2 - 4ac = 0$ ... [Given: b = a + c]
   $\Rightarrow a^2 + c^2 + 2ac - 4ac = 0$
   $\Rightarrow a^2 + c^2 - 2ac = 0$
   $\Rightarrow (a - c)^2 = 0$
   $\Rightarrow$ a - c = 0
   $\Rightarrow$ a = c
5. **(a)** 2139
   Explanation: Here, a = 5, d = (13 - 5) = 8 and l = 181
   Let the number of terms be n.
   Then, $T_n = 181$
   $\Rightarrow$ a + (n - 1)d = 181
   $\Rightarrow 5+(n-1)\times 8=181$
   $\Rightarrow$ 8n = 184
   $\Rightarrow$ n = 23
   $\therefore$ Required sum $=\frac{n}{2}(a+l)$
   $=\frac{23}{2}(5+181)=23\times 93=2139$
   Hence, the required sum is 2139
6.
   (d) $\left(-6, \frac{5}{2}\right)$
   Explanation: Distance between (0, 0) and $\left(-6, \frac{5}{2}\right)$

d = $\sqrt{(-6-0)^2+\left(\frac{5}{2}-0\right)^2}$
= $\sqrt{36+\frac{25}{4}}$
= $\sqrt{\frac{144+25}{4}}$
= $\sqrt{\frac{169}{4}}=\frac{13}{2}$ = 6.5
So, the point $\left(-6, \frac{5}{2}\right)$ does not lie in the circle.

7.

(d) -1, -3

Explanation: Given, P(x) = x^2 + 4x + 3

= x^2 + 3x + x + 3
= x(x + 3) + 1(x + 3)
= (x + 1) (x + 3)
For zeroes of polynominal (P(x) = 0
(x + 1) (x + 3) = 0
x = -1, -3

8.

(c) x = 3, y = 4.
Explanation: In triangles APQ and ATS,
∠PAQ = ∠TAS [Vertically opposite angles] ∠PQA = ∠ATS [Alternate angles]
∴ ΔAPQ∼ ΔAST [AA similarity]
$\therefore \frac{AQ}{AT}=\frac{AP}{AS}$
$\Rightarrow \frac{6}{6}=\frac{x}{3}$
$\Rightarrow x=\frac{6\times3}{6}$ = 3
And $\frac{AQ}{AT}=\frac{PQ}{ST}$
$\Rightarrow \frac{6}{6}=\frac{y}{4}$
$\Rightarrow y=\frac{4\times6}{6}$ = 4
Therefore, x = 3, y = 4

9.

(b) 30°
Explanation: In triangle POB, OP = OB [Radii of the same circle]
⇒∠BPO = ∠PBO
= 30° [Opposite angles of equal sides are equal]
Also ∠APB = 90°
[Angle in semicircle]
∴∠OPA = 90° - 30° = 60° In triangle POA, OP = OA [Radii of the same circle]
⇒∠OPA = ∠OAP = 60° [Opposite angles of equal sides are equal]
Now, OP ⊥TP, then ∠OPT = 90°
∴∠APT = 90° - 60° = 30° Also, ∠BAP + ∠PAT = 180°
⇒ 60° + ∠PAT = 180°
⇒∠PAT = 120°
Now, in triangle APT, ∠PTA + ∠APT + ∠PAT = 180°
⇒∠PTA + 30° + 120° = 180°
⇒∠PTA = 30°

10.

(b) 30 cm
Explanation: AQ = AR = 4
Similarly,
PC = CQ = 5
Similarly,
BP = BR = 6

Perimeter =AB + BC + CA

Perimeter = AR + RB + BP + PC + CQ + QA

= 4 + 6 + 6 + 5 + 5 + 4

= 30 cm

11.

(b) $\frac{p^2-1}{p^2+1}$

Explanation: Given: secθ + tanθ = p

$\Rightarrow \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$ = p

$\Rightarrow \frac{1+\sin\theta}{\cos\theta}$ = p

Squaring both sides, we get

$\Rightarrow \frac{(1+\sin\theta)^2}{\cos^2\theta}$ = p^2

$\Rightarrow \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$ = p^2

$\Rightarrow \frac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}$ = p^2

$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta}$ = p^2

$\Rightarrow$ 1 + sinθ = p^2 (1 - sinθ)

$\Rightarrow$ 1 + sinθ = p^2 - p^2 sinθ

$\Rightarrow$ sinθ + p^2 sinθ = p^2 - 1

$\Rightarrow$ sinθ(1 + p^2) = p^2 - 1

$\Rightarrow$ sinθ = $\frac{p^2-1}{p^2+1}$

12.

(c) 1 : 2

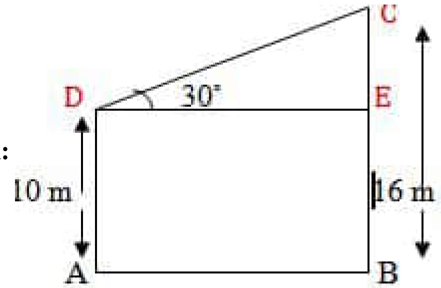
Explanation: Least composite number is 4 and the least prime number is 2.

LCM (4, 2) = 4

HCF (4, 2) = 2

The ratio of HCF to LCM = 2 : 4 or 1 : 2.

13. **(a)** 12 m

Explanation: 

Given: Two poles BC = 16 m and AD = 10 m

And $\angle \mathrm{CDE} = 30^\circ$

To find: Length of wire CD = x

$\therefore$ In triangle CDE,

$\sin 30^\circ = \frac{\mathrm{CE}}{\mathrm{CD}}$

$\Rightarrow \frac{1}{2} = \frac{\mathrm{BC-BE}}{\mathrm{CD}}$

$\Rightarrow \frac{1}{2} = \frac{16-10}{x}$

$\Rightarrow \frac{1}{2} = \frac{6}{x}$

$\Rightarrow x = 12$ m

Therefore, the length of the wire is 12 m.

14.

(b) 31.5 cm^2

Explanation: Area of quadrant = $\frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (7)^2 = \frac{77}{2}$ cm^2 = 38.5 cm^2

Area of $\triangle$BAE = $\frac{1}{2}\times$ base $\times$ height

= $\frac{1}{2}$ $\times$ AB $\times$ AE = $\frac{1}{2}$ $\times$ 7 $\times$ 2 = 7 cm^2

Hence, area of the shaded portion = Area of the quadrant ABDCA - Area of $\triangle$BAE

= (38.5 - 7) cm^2 = 31.5 cm^2

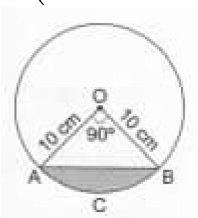
15.

(d) 28.5 cm^2

Explanation:

ar(minor segment A C B A)=ar(sector O A C B O) - $\operatorname{ar}(\Delta OAB)$

$=\left(\frac{\pi r^2\theta}{360}-\frac{1}{2}\times r\times r\right)$



$=\left(\frac{3.14\times10\times10\times90}{360}-\frac{1}{2}\times10\times10\right)\mathrm{cm}^2$

$=(78.5-50)\mathrm{cm}^2=28.5\mathrm{cm}^2$

16. **(a)** $\frac{5}{7}$

Explanation: No. of days in a leap year = 366

No. of Mondays = 52

Extra days = 366 - 52 $\times$ 7

= 366 - 364 = 2

$\therefore$ Remaining days in the week = 7 - 2 = 5

$\therefore$ Probability of being 52 Mondays in the leap year = $\frac{5}{7}$

17.

(b) $\frac{12}{25}$

Explanation: Number of multiples of 3 = 8 (3 6 9 12 15 18 21 24)

Number of multiples of 5 = 5 (5 10 15 20 25)

Number of possible outcomes (multiples of 3 or 5) = 12 (3,5,6,9,10,12,15,18,20,21,24,25)

Number of Total outcomes = 25

$\therefore$ Required Probability = $\frac{12}{25}$

18.

(c) mid points of the classes

Explanation: We know that, $d_i = x_i - a$

Where,

x_i are data or class mark and “a” is the assumed mean

i.e. d_i are the deviations of observations from assumed mean.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: For 2k + 1, 3k + 3 and 5k - 1 to form an AP

(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)

k + 2 = 2k - 4

2 + 4 = 2k - k = k

k = 6

So, both assertion and reason are correct but reason does not explain assertion.

Section B

21. In order to get the result we have to find the HCF of 2002 and 2618. There prime factors are,

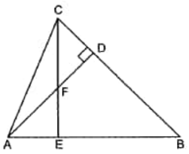
$2002 = 2 \times 7 \times 11 \times 13$ and

$2618 = 2 \times 7 \times 11 \times 17$

Hence HCF = $2 \times 7 \times 11 = 154$

22. Given Altitude AD and CE of $\triangle$ABC intersects each other at the point F.

To Prove: $\triangle FDC \sim \triangle BEC$

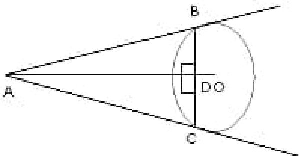


Proof: In $\triangle$'s FDC and BEC, we have

$\angle$FDC = $\angle$BEC = 90^o [$\because AD \perp BC$ and $CE \perp AB$]

$\angle$FCD = $\angle$ECB [Common angle]

Thus, by AA-criterion of similarity, we obtain $\triangle FDC \sim \triangle BEC$.

23. 

In $\triangle$ADB and $\triangle$ADC,

BD = DC

And $\angle$ADB = $\angle$ADC = 90^o

AD = AD [Common]

$\therefore \triangle$ADB $\cong \triangle$ADC [SAS]

$\therefore \angle$ABD = $\angle$ACD [By CPCT]

24. L.H.S = $\frac{\cos A}{1-\tan A} + \frac{\sin^2 A}{sinA-cosA}$

= $\frac{cos^2A}{cosA-sinA} + \frac{sin^2A}{cosA-sinA}$ [by putting tanA = $\frac{\sin A}{\cos A}$]

= $\frac{cos^2A-sin^2A}{cosA-sinA}$

= $\frac{(cosA-sinA)(cosA+sinA)}{cosA-sinA}$

= cos A + sin A

= R.H.S

OR

We have

$(3 \sin \theta + 5 \cos \theta)^2 + (5 \sin \theta - 3 \cos \theta)^2$

$= 9 (\sin^2\theta + \cos^2\theta) + 25 (\sin^2\theta + \cos^2\theta) = (9 + 25) = 34.$

Therefore, $(3 \sin \theta + 5 \cos \theta)^2 + (5 \sin \theta - 3 \cos \theta)^2 = 34$

$\Rightarrow 5^2 + (5 \sin \theta - 3 \cos \theta)^2 = 34$ [$\because 3 \sin \theta + 5 \cos \theta = 5$]

$\Rightarrow (5 \sin \theta - 3 \cos \theta) = \pm 3$ [taking square root on each side]

Hence, $(5 \sin \theta - 3 \cos \theta) = \pm 3$.

25. Area of Shaded Region

= 2 (Area of one sector ABPD) - Area of square ABCD

= $2\left(\frac{90^\circ \times \pi \times 7^2}{360^\circ}\right) - 7 \times 7$

= 28 cm^2

OR

Radius of circle (r) = OA = 7 cm.

Area of the semicircle = $\frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

= 11×7

= 77 cm^2

Area of $\triangle$ABC = $\frac{1}{2} \times$ base $\times$ height

= $\frac{1}{2} \times 14 \times 7$

= 49 cm^2

∴ Area of the shaded portion = Area of semicircle - Area of the △ABC

= 77 - 49

= 28 cm^2

Section C

26. In order to arrange the books as required, we have to find the largest number that divides 96, 240 and 336 exactly.

Clearly, such a number is their HCF.

We have,

$96 = 2^5 \times 3, 240 = 2^4 \times 3 \times 5$ and $336 = 2^4 \times 3 \times 7$

∴ HCF of 96, 240 and 336 is $2^4 \times 3 = 48$

So, there must be 48 books in each stack.

∴ Number of stacks of English books $= \frac{96}{48} = 2$

Number of stacks of Hindi books $= \frac{240}{48} = 5$

Number of stacks of Mathematics books $= \frac{336}{48} = 7$

27. Given points are A(3, -5) and B(-4, 8).

P divides AB in the ratio k:1

Using the section formula, we have:

Coordinate of point P are $\left\{\left(\frac{-4k+3}{k+1}\right)\left(\frac{8k-5}{k+1}\right)\right\}$

Now it is given, that P lies on the line x + y =0

Therefore,

$\frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$

⇒ -4k + 3 + 8k - 5 =0

⇒ -4k + 3 + 8k - 5 = 0

⇒ 4k - 2 = 0

⇒ $k = \frac{2}{4}$

⇒ $k = \frac{1}{2}$

Thus, the value of k is 1/2.

28. Assume side of one square = x m and side of other square = y m, then we have

9x = 4y + 1

⇒ $\frac{9x-1}{4}$ = y ....(i)

According to given situation we have,

$6y^2 = 29x^2 + 1$

⇒$6(\frac{9x-1}{4})^2 = 29x^2 + 1$

⇒ $\frac{3(81x^2-18x+1)}{8} = 29x^2 + 1$

⇒$243x^2 - 54x + 3 = 232x^2 + 8$

⇒$11x^2 - 54x - 5 = 0$

Factorize above quadratic equation we get

⇒(x - 5)(11x + 1) = 0

⇒x = 5 or x = $\frac{-1}{11}$ (negative value is rejected)

∴x = 5m

When x = 5, then y = $\frac{9\times5-1}{4}$= 11m (From (i))

Hence sides of the square are 5m and 11m.

OR

Given, $x^2 + 5x - (a^2 + a - 6) = 0$

splitting $a^2 + a - 6$

⇒ $x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

⇒ x^2 + 5x - [a(a + 3) - 2 (a + 3)] = 0

⇒ x^2 + 5x - (a + 3)(a - 2) = 0

Now splitting the middle term

$\Rightarrow x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$

$\Rightarrow x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$\Rightarrow [x + (a + 3)][x - (a - 2)] = 0$

$\Rightarrow x + (a + 3) = 0$ or $x - (a - 2) = 0$

Therefore, $x = -(a + 3)\ or\ (a - 2)$

29. We know that the lengths of tangents from an exterior point to a circle are equal.

AP = AR, ...(i) [tangents from A]

BP = BQ, ... (ii) [tangents from B]

CQ = CR. ...(iii) [tangents from C]

a. AB + CQ = AP + BP + CQ

= AR + BQ + CR [using (i), (ii) and (iii)]

= (AR + CR) + BQ = AC + BQ.

b. Join OA, OB and OC.

Area $(\triangle ABC)$ = area $(\triangle OAB)$

+ area (ΔOBC)

+ area $(\triangle OCA)$

$= \left(\frac{1}{2} \times AB \times OP\right)$

$+ \left(\frac{1}{2} \times BC \times OQ\right)$

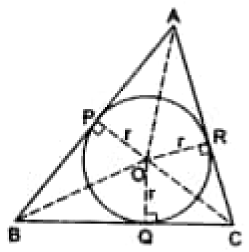
$+ \left(\frac{1}{2} \times CA \times OR\right))$

$= \left(\frac{1}{2} \times AB \times r\right) + \left(\frac{1}{2} \times BC \times r\right) + \left(\frac{1}{2} \times CA \times r\right)$

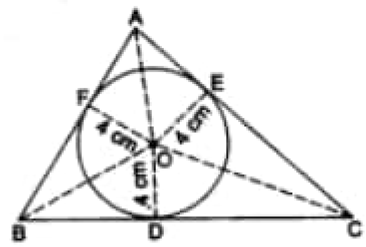
$= \frac{1}{2}(AB + BC + CA) \times r$

$= \frac{1}{2}(AB + BC + CA) \times r$

$= \frac{1}{2}(\text{ perimeter of } \Delta ABC) \times r$



OR



We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$AE = AF = x\ cm,$

$BD = BF = 6cm, CD = CE = 8cm.$

so, AB = AF+ BF = (x + 6) cm,

BC = BD + CD = 14 cm,

AC = CE+AE = (x+8) cm.

Perimeter, 2s = AB + BC+AC

= [(x + 6) +14 + (x+8)] cm

= (2x + 28) cm

$\Rightarrow$ s = (x +14) cm.

$\therefore \quad \text{ar}(\Delta ABC) = \sqrt{s(s - AB)(s - BC)(s - AC)}$

$= \sqrt{(x + 14)\{(x + 14) - (x + 6)\}\{(x + 14) - 14\}\{(x + 14) - (x + 8)\}}\text{cm}^2$

$= \sqrt{48x(x + 14)}\text{cm}^2$. ...(i)

Join OE and OF and also OA, OB and OC.

$\therefore \quad \operatorname{ar}(\Delta ABC)$

$= \operatorname{ar}(\triangle OAB) + \operatorname{ar}(\triangle OBC) + \operatorname{ar}(\triangle OCA)$

$= \left(\frac{1}{2} \times AB \times OF\right) + \left(\frac{1}{2} \times BC \times OD\right) + \left(\frac{1}{2} \times AC \times OE\right)$

$= \left[\frac{1}{2} \times (x+6) \times 4\right] + \left[\frac{1}{2} \times 14 \times 4\right] + \left[\frac{1}{2} \times (x+8) \times 4\right]$

$= 2[(x+6) + 14 + (x+8)]$

$= 4(x+14)\text{cm}^2$. ...(ii)

From (i) and (ii), we get

$\sqrt{48x(x+14)} = 4(x+14)$

$\Rightarrow \quad 48x(x+14) = 16(x+14)^2$

$\Rightarrow \quad 48x = 16(x+14)$

$\Rightarrow x = \frac{16 \times 14}{32} = 7$

$\therefore \quad AB = (x+6)\text{cm}$

$= (7+6)\text{cm}$

$= 13cm$

and

$AC = (x+8)cm$

$= (7+8)cm$

$= 15cm$

30. We have,

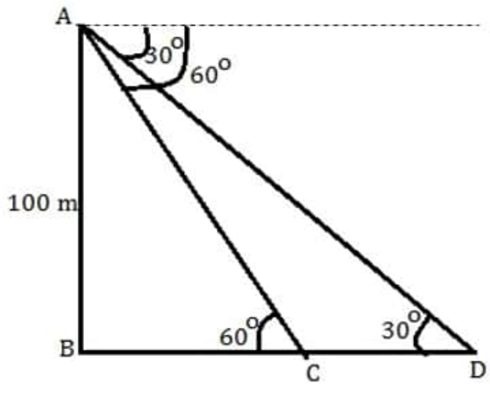
$\text{LHS} = \frac{\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)}{\left(\frac{1}{\cos^3 A}-\frac{1}{\sin^3 A}\right)}$

$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{\cos^2 A+\sin^2 A}{\sin A \cos A}\right)(\sin A-\cos A)}{\left(\frac{\sin^3 A-\cos^3 A}{\sin^3 A \cos^3 A}\right)}$

$\Rightarrow \quad \text{LHS} = \frac{\left(1+\frac{1}{\sin A \cos A}\right)(\sin A-\cos A) \sin^3 A \cos^3 A}{(\sin^3 A-\cos^3 A)}$

$\Rightarrow \quad \text{LHS} = \frac{(\sin A \cos A+1)(\sin A-\cos A) \sin^2 A \cos^2 A}{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A \cos A)}$ $[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$

$\Rightarrow \quad \text{LHS} = \frac{(\sin A \cos A+1) \sin^2 A \cos^2 A}{(1+\sin A \cos A)}$ $= \sin^2 A \cos^2 A = \text{RHS}$

31. 

Height of the tower = 100 m

Let BC = x and BD = y

Consider the $\triangle ABC$,

$\frac{\text{AB}}{\text{BC}} = \tan 60^\circ$

$\Rightarrow \frac{100}{\text{x}} = \sqrt{3}$

$\Rightarrow \text{x} = \frac{100}{\sqrt{3}}\text{m}$

Consider the $\triangle ABD$,

$\frac{\text{AB}}{\text{BD}} = \tan 30^\circ$

$\frac{1}{\sqrt{3}} = \frac{100}{y}$

$y = 100\sqrt{3}$

We know that ,

BD = BC + CD

y = x + CD

CD = y - x

$= 100\sqrt{3} - \frac{100}{\sqrt{3}}$

$= \frac{200}{\sqrt{3}}\text{m}$

$= \frac{200\sqrt{3}}{3}\text{m}$

$= 115.466\text{m}$

Section D

32. Let the fixed charge be Rs. x and additional charge by Rs. y.

According to question,

x + (7 - 3)y = 27

or x + 4y = 27 ....(i)

and x + (5 - 3)y = 21

x + 2y = 21 ....(ii)

On substracting (i) and (ii), we get

2y = 6

y=3

putting y in (i),

x + 4(3) = 27

x = 15

$\therefore$ x = Rs. 15 and y = Rs. 3

$\therefore$ Fixed charge = Rs. 15

$\therefore$ Charge for each extra day = Rs. 3

OR

Let the actual speed of the train be x km/hr and the actual time taken be y hours. Then,

Distance covered = (xy) km ... (i) [$\because$ Distance = Speed $\times$ Time]

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e., when speed is (x + 6) km/hr, time of journey is (y - 4) hours.

$\therefore$ Distance covered = (x + 6) (y - 4)

$\Rightarrow$ xy = (x + 6) (y - 4) [Using (i)]

$\Rightarrow$ -4x + 6y - 24 = 0

$\Rightarrow$ -2.x + 3y - 12 = 0 ... (ii)

When the speed is reduced by 6 km/hr, then the time of journey is increased by 6 hours i.e., when speed is (x - 6) km/hr, time of journey is (y + 6) hours.

$\therefore$ Distance covered = (x - 6) (y + 6)

$\Rightarrow$ xy = (x - 6) (y + 6) [Using (i)]

$\Rightarrow$ 6x - 6y - 36 = 0

$\Rightarrow$ x - y - 6 = 0 ... (iii)

Thus, we obtain the following system of equations:

- 2x + 3y - 12 = 0

x-y- 6 = 0

By using cross-multiplication, we have,

$\frac{x}{3\times-6-(-1)\times-12} = \frac{-y}{-2\times-6-1\times-12} = \frac{1}{-2\times-1-1\times3}$

$\Rightarrow \frac{x}{-30} = \frac{-y}{24} = \frac{1}{-1}$

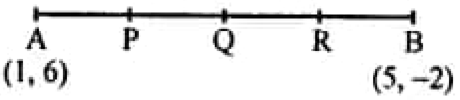
$\Rightarrow$ x = 30 and y = 24.

Putting the values of x and y in equation (i), we obtain

Distance = (30 $\times$ 24) km = 720 km.

Hence, the length of the journey is 720 km.

33. Points P, Q and R in order divide a line segment joining the points A(1, 6) and B(5, -2) in 4 equal parts.



P divides AB in the ratio of 1:3 Let coordinates of P be (x, y), then

$x=\frac{mx_2+nx_1}{m+n}=\frac{1\times5+3\times1}{1+3}$
$=\frac{5+3}{4}=\frac{8}{4}=2$
$y=\frac{my_2+ny_1}{m+n}=\frac{1\times(-2)+3\times6}{1+3}$
$=\frac{-2+18}{4}=\frac{16}{4}=4$

$\therefore$ Coordinates of P are (2, 4)

Similarly,

Q divides AB in 2:2 or 1:1 and Q is midpoint of AB.

$\therefore$ Coordinates of Q will be $\left(\frac{1+5}{2},\frac{6-2}{2}\right)$

or $\left(\frac{6}{2},\frac{4}{2}\right)$ or (3, 2)

and R divides AB in the ratio of 3:1

Coordinates of R will be
$\left(\frac{3\times5+1\times1}{3+1},\frac{3\times(-2)+1\times6}{3+1}\right)$
or $\left(\frac{15+1}{4},\frac{-6+6}{4}\right)$ or $\left(\frac{16}{4},\frac{0}{4}\right)$ or $(4,0)$

34. a. Total surface area of block

= TSA of cube + CSA of hemisphere - Base area of hemisphere

= $6a^2 + 2\pi r^2 - \pi r^2$

= $6a^2 + \pi r^2$

= $\left(6\times6^2+\frac{22}{7}\times2.1\times2.1\right)\text{cm}^2$

= (216 + 13.86) cm^2

= 229.86 cm^2

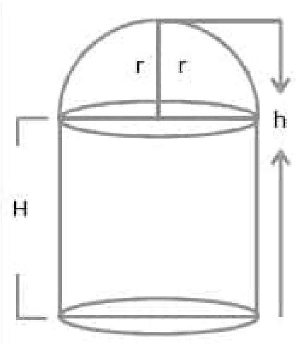
b. Volume of block

= $6^3+\frac{2}{3}\times\frac{22}{7}\times(2.1)^3$

= (216 + 19.40) cm^3

= 235.40 cm^3

OR



Let the radius of the hemispherical dome be r and the total height of the building be h.

Since, the base diameter of the dome is equal to $\frac{2}{3}$ of the total height

$2r=\frac{2}{3}h$
$\Rightarrow r=\frac{h}{3}$

Let H be the height of the cylindrical position.

$\Rightarrow H=h-r=h-\frac{h}{3}=\frac{2h}{3}$

Volume of air inside the building = Volume of air inside the dome + Volume of air inside the cylinder

$\Rightarrow 67\frac{1}{21}=\frac{2}{3}\pi r^3+\pi r^2H$
$\Rightarrow \frac{1408}{21}=\pi r^2\left(\frac{2}{3}r+H\right)$
$\Rightarrow \frac{1408}{21}=\frac{22}{7}\times\left(\frac{h}{3}\right)^2\left(\frac{2}{3}\times\frac{h}{3}+\frac{2h}{3}\right)$
$\Rightarrow \frac{1408\times7}{22\times21}=\frac{h^2}{9}\times\left(\frac{2h}{9}+\frac{2h}{3}\right)$
$\Rightarrow \frac{64}{3}=\frac{h^2}{9}\times\left(\frac{8h}{9}\right)$
$\Rightarrow \frac{64\times9\times9}{3\times8}=h^3$

$\Rightarrow h^3 = 8 \times 27$
$\Rightarrow h = 6$
Thus, the height of the building is 6 m.

35. It is given that, Sum of first p terms of an AP = q
and Sum of the first q terms the same AP = p
Let us take the first term as a and the common difference d
Therefore, the sum $S_n = \frac{n}{2}[2a + (n-1)d]$
$q = \frac{p}{2}[2a + (p-1)d]$
$p = \frac{q}{2}[2a + (q-1)d]$
Subtracting the sum of the q terms from the sum of p terms
we get
q - p = $\left[\frac{p}{2}(2a + (p-1)d] - \frac{q}{2}[2a + (q-1)d\right]$
q - p = $a(p-q) + \frac{d}{2}\left(p^2 - p - q^2 + q\right)$
After solving the equation we get
$d = -\frac{2(p+q)}{pq}$
Now with $d = -\frac{2(p+q)}{pq}$, the first term of the series is a and the number of terms is (p + q)
$S_n = \frac{n}{2}[2a + (n-1)d]$
Sp + q = $\frac{p+q}{2}[2a + (p+q-1)d] = \frac{p+q}{pq}(-pq)$
Therefore, the sum is -(p + q).

Section E

36. i. Zeroes are -2 and 8
$\alpha + \beta$ = -2 + 8 = 6
$\alpha\beta$ =-2 $\times$ 8 = -16
expression of polynomial
$x^2 - (\alpha + \beta)x + \alpha\beta$
$x^2 - 6x - 16$

ii. $P(x) = x^2 - 6x - 16$
$P(4) = 4^2 - 6(4) - 16$
= 16 - 24 - 16
= -24

iii. $P(x) = -x^2 + 3x - 2$
$\alpha + \beta = \frac{-3}{-1}$
$\alpha + \beta$ = 3 ...(i)
$\alpha\beta = \frac{-2}{-1}$
$\alpha\beta$ = 2 ...(ii)
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$(\alpha - \beta)^2 = (3)^2 - 4(2)$
$(\alpha - \beta)^2 = 9 - 8$
$\alpha - \beta = \pm\sqrt{1}$
$\alpha - \beta = \pm 1$
Taking
$\alpha - \beta = 1$
$\alpha + \beta = 3$
$2\alpha = 4$
$\alpha = 2$
Put α = 2 in, $\alpha - \beta = 1$
$2 - \beta = 1$
$\beta = 1$

OR

$\alpha + \beta = \frac{-3}{-1} = 3$
$\alpha\beta = \frac{-2}{-1} = 2$

37. i. Median class : 100 - 110

ii. No. of leaves equal to or more than 10cm(100 mm) = 23

a.

C.I	f	cf
70-80	3	3
80-90	5	8
90-100	9	17
100-110	12	29
110-120	5	34
120-130	4	38
130-140	2	40 = N

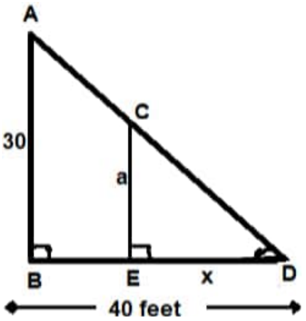
Median $= 100 + \frac{10}{12}(20 - 17) = 102 \cdot 5$

OR

b. Modal class is 100 - 110

Mode $= 100 + 10 \times \frac{12-9}{24-9-5} = 103$

38. i.



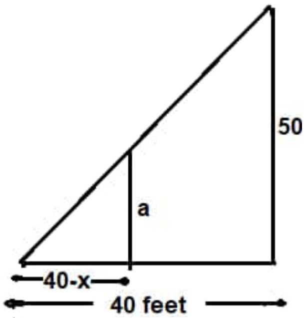
$\triangle$ABD $\sim$ $\triangle$CED (by AA criteria)

$\frac{30}{a} = \frac{40}{x}$

$\frac{x}{a} = \frac{40}{30}$

a = $\frac{30}{40}x$

Again



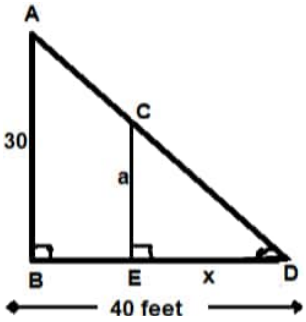
$\frac{40-x}{40} = \frac{a}{50}$

$\frac{40-x}{40} = \frac{30 \times x}{40 \times 50}$

8000 - 200x = 120x

8000 = 320x

$\therefore$ x = 25 feet

ii. 

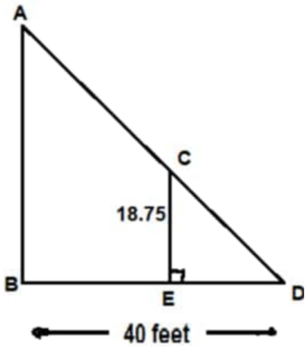
$\frac{x}{40} = \frac{a}{30}$

$\frac{25}{40} = \frac{a}{30}$

$\frac{25 \times 30}{40} = a$

$\frac{75}{4}$ = a

a = 18.75 feet

iii. 

AD = $\sqrt{30^2 + 40^2}$

$= \sqrt{900 + 1600}$

$= \sqrt{2500}$

AD = 50 feet

In $\triangle$CED

CD = $\sqrt{18.75^2 + 25^2}$

$= \sqrt{976.5625}$

= 31.25 feet

AC = AD - CD

= 50 - 31.25

= 18.75 feet

OR

$\sqrt{40^2 + 20^2}$

$= \sqrt{1600 + 400}$

$= \sqrt{2000}$

$= 20\sqrt{5}$ feet