Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, then the value of x is **[1]**

a) 0 b) 7

c) 3 d) 10

2. If A $=\begin{bmatrix}\mathrm{a} & 0 & 0\\0 & \mathrm{a} & 0\\0 & 0 & a\end{bmatrix}$, then det (adj A) equals **[1]**

a) a^{27} b) a^2

c) a^6 d) a^9

3. The system of equations, x + y + z = 1, 3x + 6y + z = 8, αx + 2y + 3z = 1 has a unique solution for **[1]**

a) α not equal to 0 b) all rational α

c) all real α d) all integral α

4. If f(x) = x^n, then the value of f(1) - $\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}\ldots+\frac{(-1)^n f^n(1)}{n!}$ is **[1]**

a) 1 b) 0

c) 2^n d) 2

[1]

5. The direction ratios of the line perpendicular to the lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$ and $\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$ are proportional to

a) 4, 5, 7 b) 4, -5, 7

c) -4, 5, 7 d) 4, -5, -7

6. Solution of $\left(x^2 - y^2\right)\ dx + 2xy\,dy = 0$ is **[1]**

a) $x^2+y^3=Cx$ b) $x^2-y^2=Cx$

c) $x^2+y^2=Cx$ d) $x^3+y^2=Cx$

7. The feasible region for an LPP is always a **[1]**

a) type of polygon b) convex polygon

c) concave polygon d) Straight line

8. If β is perpendicular to both α and γ, where α = $\hat{k}$ and γ = $\gamma = 2\hat{i}+3\hat{j}+4\hat{k}$, then what is β equal to? **[1]**

a) $3\hat{i}+2\hat{j}$ b) $-3\hat{i}+2\hat{j}$

c) $-2\hat{i}+3\hat{j}$ d) $2\hat{i}-3\hat{j}$

9. $\int_0^1 \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}dx$ is equal to **[1]**

a) $\frac{\pi}{2}$ b) 0

c) π d) $\frac{\pi}{4}$

10. If A + B = $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$ and A – 2 B = $\begin{bmatrix}-1 & 1\\1 & -1\end{bmatrix}$, then A = ? **[1]**

a) $\frac{1}{3}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$ b) $\frac{1}{3}\begin{bmatrix}1 & 1\\3 & 1\end{bmatrix}$

c) $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ d) $\frac{1}{2}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$

11. The point which does not lie in the half plane 2x + 3y - 12 $\leq$ 0 is **[1]**

a) (2,1) b) (2, 3)

c) (1, 2) d) (-3, 2)

12. For any vector α, what is $(\alpha\cdot\hat{\mathbf{i}})\hat{\mathbf{i}}+(\alpha\cdot\hat{\mathbf{j}})\hat{\mathbf{j}}+(\alpha\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}$ equal to? **[1]**

a) -α b) α

c) 0 d) 3α

13. The sum of products of elements of any row with the cofactors of corresponding elements is equal to **[1]**

a) a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13} b) a_{11} A_{11} + a_{12} A_{12} + a_{21} A_{13}

c) a_{11} A_{11} + a_{12} A_{13} + a_{13} A_{12} d) a_{12} A_{21} + a_{12} A_{21} + a_{21} A_{13}

14. If $P(A)=\frac{3}{10}, P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$, then $P(B/A)+P(A/B)$ equals **[1]**

a) $\frac{7}{12}$ b) $\frac{5}{12}$

c) $\frac{1}{3}$ d) $\frac{1}{4}$

15. The general solution of a differential equation of the type $\frac{dx}{dy}+\mathrm{P}_1 x=\mathrm{Q}_1$ is **[1]**

a) $xe^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

b) $ye^{\int P_1 dy} = \int \left(Q_1 e^{\int P_1 dy}\right) dy + C$

c) $xe^{\int P^1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

d) $y \cdot e^{\int P\, dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

16. If $\vec{a} \cdot \hat{i} = \vec{a} \cdot (\hat{i} + \hat{j}) = \vec{a} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$, then $\vec{a}$ is **[1]**

a) $\hat{k}$

b) $\hat{j}$

c) $\hat{i} + \hat{j} + \hat{k}$

d) $\hat{i}$

17. For what value of k may the function f(x) = $\begin{cases} k\left(3x^2 - 5x\right), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ become continuous? **[1]**

a) 1

b) $-\frac{1}{2}$

c) 0

d) No value

18. If a line makes an angle of 30^o with the positive direction of x-axis, 120^o with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is: **[1]**

a) 0^o

b) 120^o

c) 90^o

d) 60^o

19. **Assertion (A):** The average rate of change of the function y = 15 - x^2 between x = 2 and x = 3 is -5. **[1]**
Reason (R): Average rate of change $\delta_y = y_{\text{at } x = 3} - y_{\text{at } x = 2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Let A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}, then f is a bijective function. **[1]**
Reason (R): Let A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}, then f is a surjective function.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the interval for the principal value of function and draw its graph: $\cot^{-1}$ x. **[2]**

OR

Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

22. Show that f(x) = x^3 - 15x^2 + 75x - 50 is increasing on R. **[2]**

23. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm. **[2]**

OR

Is the function $cos3x$ decreasing on $\left(0, \frac{\pi}{2}\right)$?

24. Evaluate: $\int x^2 \cos x dx$. **[2]**

25. Find the maximum and minimum value, f(x) = |x + 2| - 1 **[2]**

Section C

26. Find $\int \frac{5x-2}{1+2x+3x^2} dx$. **[3]**

27. Probabilities of solving a specific problem independently by A and Bare $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to **[3]**

solve problem independently, then find the probability that

i. problem is solved.

ii. exactly one of them solves the problem.

28. If f(2a - x) = -f(x), prove that $\int_0^{2a} f(x)dx = 0$ **[3]**

OR

Evaluate: $\int e^{\sin x} \sin 2x\, dx$

29. Find the particular solution of the differential equation $x\frac{dy}{dx} + y + \frac{1}{1+x^2}$ = 0, given that y(1) = 0. **[3]**

OR

Find the general solution of the differential equation: $\left(1 - x^2\right)\frac{dy}{dx} + xy = ax$

30. Solve the Linear Programming Problem graphically: **[3]**

Minimize Z = 3 x_1 + 5x_2

Subject to

$x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

OR

Solve the Linear Programming Problem graphically:

Minimize Z = l8x + 10y

Subject to

4x + y > 20

2x + 3 y > 30

x, y > 0

31. Find all points of discontinuity of f where f is defined as follows,f(x) = $\begin{cases} |x| + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$ **[3]**

Section D

32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. **[5]**

33. If A = {1, 2, 3, 4}, define relations on A which have properties of being: **[5]**

a. reflexive, transitive but not symmetric

b. symmetric but neither reflexive nor transitive

c. reflexive, symmetric and transitive.

OR

Let A = [-1, 1]. Then, discuss whether the following functions defined on A are one-one, onto or bijective:

i. $f(x) = \frac{x}{2}$

ii. g(x) = |x|

iii. h(x) = x|x|

iv. k(x) = x^2

34. Solve the system of the following equations: (Using matrices): **[5]**

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4; \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1; \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2;$

35. Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes **[5]**

$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

OR

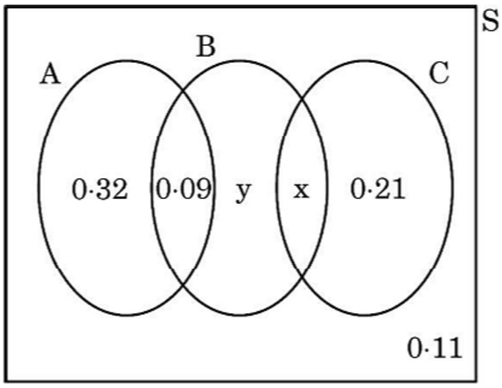
Find the shortest distance between the given lines. $\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$, $\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.



i. Find the value of x. (1)

ii. Find the value of y. (1)

iii. Find $P\left(\frac{C}{B}\right)$. (2)

OR

Find the probability that a randomly selected person of the society does Yoga of type A or B but not C. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

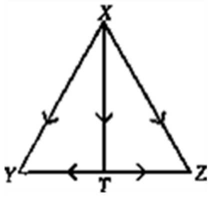
i. If $\vec{p}, \vec{q}, \vec{r}$ are the vectors represented by the sides of a triangle taken in order, then find $\vec{q} + \vec{r}$. (1)

ii. If ABCD is a parallelogram and AC and BD are its diagonals, then find the value of $\overrightarrow{AC} + \overrightarrow{BD}$. (1)

iii. If ABCD is a parallelogram, where $\overrightarrow{AB}$ = $2\vec{a}$ and $\overrightarrow{BC}$ = $2\vec{b}$, then find the value of $\overrightarrow{AC} - \overrightarrow{BD}$. (2)

OR

If T is the mid point of side YZ of $\triangle$XYZ, then what is the value of $\overrightarrow{XY} + \overrightarrow{XZ}$. (2)



38. **Read the following text carefully and answer the questions that follow:** **[4]**

In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.

i. If l(x) denotes the combined light intensity, then find the value of x so that I(x) is minimum. (1)

ii. Find the darkest spot between the two lights. (1)

iii. If you are in between the lamp posts, at distance x feet from the stronger light, then write the combined light intensity coming from both lamp posts as function of x. (2)

OR

Find the minimum combined light intensity? (2)

Solution

Section A

1.

(c) 3

Explanation:

Given, $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2x\\3x\end{bmatrix}+\begin{bmatrix}-y\\y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix} \Rightarrow \begin{bmatrix}2x-y\\3x+y\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$

$\Rightarrow$ 2x - y = 10 ...(i)

and 3x + y = 5 ...(ii)

On solving Eqs. (i) and (ii), we get x = 3

2.

(c) a^6

Explanation:

$A=\begin{bmatrix}a&0&0\\0&a&0\\0&0&a\end{bmatrix}$

$|A| = a^3$

$|adj\ A| = |A|^{3-1} = |A|$

$|adj\ A| = (a^3)^2 = a^6$

3.

(d) all integral α

Explanation:

The given system of equations has unique solution, if:

$\begin{vmatrix}1&1&1\\3&6&1\\\alpha&2&3\end{vmatrix}\neq 0$

$\Rightarrow$ 1(18 - 2) - 1(9 - α) + 1(6 - 6α) $\neq$ 0

$\Rightarrow$ 13 - 5α $\neq$ 0

$\Rightarrow \alpha \neq \frac{13}{5}$. (Since α is not integral value)

Thus, unique solution exists for all integral values of α.

4.

(b) 0

Explanation:

0

5. **(a)** 4, 5, 7

Explanation:

We have,

$\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{1}$

$\frac{x+5}{1}=\frac{y+3}{2}=\frac{z-4}{-2}$

The direction ratios of the given lines are proportional to 2, -3, 1 and 1, 2, -2.

The vectors parallel to the given lines are $\overrightarrow{b_1} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\overrightarrow{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

Vector perpendicular to the vectors b_1 & b_2 is ,

$\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$

= $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$

= $4\hat{i} + 5\hat{j} + 7\hat{k}$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4, 5, 7.

6.

(c) $x^2 + y^2 = Cx$

Explanation:

$(x^2 - y^2)dx + 2xydy = 0 \Rightarrow \frac{dy}{dx} = \frac{y^3 - x^3}{2xy}$

Put y = vx, we have; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2xvx} \Rightarrow x\frac{dv}{dx} = -\left(\frac{v^2+1}{2v}\right)$

$\Rightarrow \int \frac{2v}{v^2+1}dv = -\int \frac{1}{x}dx \Rightarrow \log|v^2 + 1| = -\log|x| + \log C$

$\Rightarrow \log|v^2 + 1| + \log|x| = \log C \Rightarrow (v^2 + 1)|x| = C \Rightarrow (x^2 + y^2) = Cx$

7.

(b) convex polygon

Explanation:

Feasible region for an LPP is always a convex polygon.

8.

(b) $-3\hat{i} + 2\hat{j}$

Explanation:

Given that, $\alpha = \hat{k}$

and $\gamma = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Since, β is perpendicular to both α and γ.

i.e., $\beta = \pm(\alpha \times \gamma) = \pm\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$

= $\pm\hat{i}(0 - 3) - \hat{j}(0 - 2) + \hat{k}(0 - 0)$

= $\pm(-3\hat{i} + 2\hat{j})$

9. **(a)** $\frac{\pi}{2}$

Explanation:

We have,

$I = \int_0^1 \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}dx$

We know since $\int f'(x) = f(x)$

$f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $f'(x) = \frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}$

Therefore, $I = \left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1$

= $\sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2}$

10.

(b) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

Explanation:

A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ ...(i)

A - 2B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ ...(ii)

adding 2 $\times$ (i) and (ii), we get

2A + 2B = $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ ...(iii)

A - 2B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ ...(iv)

adding (iii) and (iv), we get

$\Rightarrow$ 3A = $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$\Rightarrow$ A = $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

11.

(b) (2, 3)

Explanation:

Since (2, 3) does not satisfy 2x + 3y - 12 $\leq$ 0 as 2 $\times$ 2 + 3 $\times$ 3 - 12 = 4 + 9 - 12 = 1 $\neq$ 0

12.

(b) α

Explanation:

Let $\alpha = x\hat{i} + y\hat{j} + z\hat{k}$

Then $(\alpha \cdot \hat{i})\hat{i} + (\alpha \cdot \hat{j})\hat{j} + (\alpha \cdot \hat{k})\hat{k}$

= $\{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{i}\}\hat{i}$ + $\{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{j}\}\hat{j}$ + $\{(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{k}\}\hat{k}$

= $(x)\hat{i} + (y)\hat{j} + (z)\hat{k} = \alpha$

13. **(a)** a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}

Explanation:

By the definition of expansion of determinant, the required relation is

a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}

14. **(a)** $\frac{7}{12}$

Explanation:

Here, $P(A) = \frac{3}{10}, P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$

$P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$

$= \frac{P(A)+P(B)-P(A \cup B)}{P(A)} + \frac{P(A)+P(B)-P(A \cup B)}{P(B)}$

$= \frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{3}{10}} + \frac{\frac{3}{10}+\frac{2}{5}-\frac{3}{5}}{\frac{2}{5}}$

$= \frac{\frac{1}{10}}{\frac{3}{10}} + \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

15. **(a)** $xe^{\int \mathrm{P}_1 dy} = \int \left(\mathrm{Q}_1 e^{\int \mathrm{P}_1 dy}\right) dy + \mathrm{C}$

Explanation:

The integrating factor of the given differential equation

$\frac{\mathrm{dx}}{\mathrm{dy}} + \mathrm{P}_1\mathrm{x} = \mathrm{Q}_1$ is $\mathrm{e}^{\int \mathrm{P}_1 \mathrm{dy}}$

Thus, the general solution of the differential equation is given by,

$\mathrm{x}(\mathrm{I.F.}) = \int(\mathrm{Q}_1 \times \mathrm{I.F.})\mathrm{dy} + \mathrm{C}$

$\Rightarrow \mathrm{x.e}^{\int \mathrm{P}_1 \mathrm{dy}} = \int \left(\mathrm{Q}_1 \mathrm{e}^{\int \mathrm{P}_1 \mathrm{dy}}\right) \mathrm{dy} + \mathrm{C}$

16.

(d) $\hat{\mathrm{i}}$

Explanation:

$\hat{i}$

17\.

(d) No value

Explanation:

No value

18\.

(c) 90^o

Explanation:

To find the angle with the z-axis, we use the relation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

where $\alpha = 30^\circ$ and $\beta = 120^\circ$. Calculating, we get:

$\cos^2 30^\circ = \frac{3}{4}, \cos^2 120^\circ = \frac{1}{4}$

Thus,

$\frac{3}{4} + \frac{1}{4} + \cos^2\gamma = 1 \implies \cos^2\gamma = 0 \implies \gamma = 90^\circ.$

So, the angle with the z-axis is 90^o.

19\. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Let y = f(x) = 15 - x^2

If x changes from 2 to 3 then δ_x = 3 - 2 = 1

Again f(3) = 15 - 9 = 16 and f(2) = 15 - 4 = 11

Therefore δ_y = f(3) - f(2) = 6 - 11 = -5

20\.

(d) A is false but R is true.

Explanation:

We have, A = {1, 5, 8, 9}, B = {4, 6} and f = {(1, 4), (5, 6), (8, 4), (9, 6)}

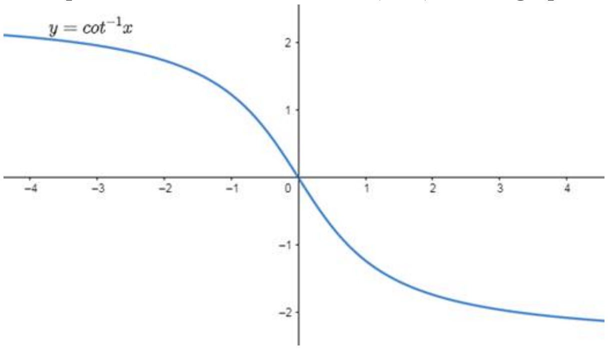
So, all elements of B has a domain element on A or we can say elements 1 and 8 & 5 and 9 have some range 4 & 6, respectively.

Therefore, f : A $\rightarrow$ B is a surjective function not one to one function.

Also, for a bijective function, f must be both one to one onto.

Section B

21\. Principal value branch of $\cot^{-1}$ x is (0, π) and its graph is shown below.



OR

We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right].$

$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1).$

$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\left[\begin{array}{c} \because \tan^{-1}(\tan x) = x, x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0,\ \pi) \\ and\ \tan^{-1}(-x) = -\tan^{-1}x \end{array}\right]$

$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi+4\pi-3\pi}{12}$

$= \frac{-5\pi+4\pi}{12} = \frac{-\pi}{12}$

22. Consider the given function,

f(x) = x^3 -15x^2 + 75x - 50

Domain of the function is R.

f’(x) = 3x^2 - 30x + 75

= 3(x^2 - 10 + 25)

= 3(x - 5) (x - 5)

= 3$(x - 5)^2$

f’(x) = 0 for x = 5

for x < 5

f’(x) > 5

and

for x > 5

f’(x) > 5

we can see throughout R the derivative is +ve but at x = 5 it is 0 so it is increasing.

23. Let r be the radius of sphere and V be its volume.

Then V = $\frac{4}{3}\pi r^3$........(i)

Given, $\frac{dV}{dt}$ = 3 cm^3/s

Differentiating (i) both sides w.r.t x,we get,

$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$

$\Rightarrow \quad 3 = \frac{4}{3}(3\pi r^2)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi r^2}$ ....(ii)

Now, let S be the surface area of sphere, then

S = $4\pi r^2$

$\Rightarrow \quad \frac{dS}{dt} = 4\pi(2r)\frac{dr}{dt}$

$\Rightarrow \quad \frac{dS}{dt} = 8\pi r\left(\frac{3}{4\pi r^2}\right)$ [using Eq.(ii)]

$\Rightarrow \quad \left(\frac{dS}{dt}\right) = \frac{6}{r}$

when r = 2, then

$\frac{dS}{dt} = \frac{6}{2}$ = 3 cm^2/s

Therefore,the rate of inrcrease of the surface area of sphere is 3 cm^2/s when it's radius is 2 cm.

OR

Let f(x) = cos 3x

$\therefore$ f'(x) = -3 sin 3x

Now, $f'(x)$ = 0

$\Rightarrow$ sin 3x = 0

$\Rightarrow$ 3x = π, as $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0, \frac{\pi}{2}\right)$ into two distinct intervals.

i.e. $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Now, in the interval, $\left(0, \frac{\pi}{3}\right)$

$f'(x) = -3\sin 3x < 0$ as $\left(0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right)$

Therefore, 'f ' is strictly decreasing in interval $\left(0, \frac{\pi}{3}\right)$

Now, in the interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$f'(x) = -3\sin 3x > 0$ as $\frac{\pi}{3} < \mathrm{x} < \frac{\pi}{2} \Rightarrow \pi < 3\mathrm{x} < \frac{3\pi}{2}$

Therefore, 'f ' is strictly increasing in the interval $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

24. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$\int \mathrm{a.b} \cdot \mathrm{dx} = \mathrm{a} \int \mathrm{bdx} - \int \left[\frac{\mathrm{da}}{\mathrm{dx}} \cdot \int \mathrm{bdx}\right] \mathrm{dx}$

$\Rightarrow \int x^2 \cos x\, dx = x^2 \int \cos x\, dx - \int \left[\frac{dx^2}{dx} \cdot \int \cos x\, dx\right] dx$

$= x^2 \sin x - \int [2x \times \sin x]\, dx$

$= x^2 \sin x - 2[\int x \sin x\, dx]$

Again applying by the part method in the second half, we get

$x^2 \sin x - 2 \int x \sin x\, dx$

$= x^2 \sin x - 2\left[x \int \sin x\, dx - \int \left(\frac{dx}{dx} \cdot \int \sin x\, dx\right) dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int 1.(-\cos x) dx\right]$

$= x^2 \sin x - 2[-x\cos x + \sin x] + c$

$= x^2 \sin x + 2x\cos x - 2\sin x + c$

25. It is given that f(x) = |x + 2| - 1

Now, we can see that $|x + 2| \geq 0$ for every $x \in R$

$\Rightarrow f(x) = |x + 2| - 1 \geq -1$ for every $x \in R$

Clearly, the minimum value of f is attained when |x + 2| = 0

i.e, |x + 2| = 0

$\Rightarrow$ x = -2

Then, Minimum value of f = f(-2) = |-2 + 2| - 1 = -1

Therefore, function f does not have a maximum value.

Section C

26. According to the question, $I = \int \frac{5x-2}{1+2x+3x^2} dx$

$(5x - 2)$ can be written as ,

$5x - 2 = A\frac{d}{dx}\left(1 + 2x + 3x^2\right) + B$

$I = \int \frac{5x-2}{1+2x+3x^2} dx = \int \frac{A\frac{d}{dx}\left(1+2x+3x^2\right)+B}{1+2x+3x^2} dx$ ...(i)

$\Rightarrow 5x - 2 = A(2 + 6x) + B$

Comparing the coefficients of x and constant terms,

$5 = 6A \Rightarrow A = \frac{5}{6}$

and $-2 = 2A + B \Rightarrow B = -2A - 2$

$= -\frac{5}{3} - 2 = -\frac{11}{3}$ $\left[\because A = \frac{5}{6}\right]$

From Eq. (i), we get

$I = \int \frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2} dx$

$\Rightarrow \quad I = \int \frac{\frac{5}{6}(2+6x)}{1+2x+3x^2} dx - \int \frac{\left(\frac{11}{3}\right)}{1+2x+3x^2} dx$

$\Rightarrow I = I_1 - I_2$ ...(ii)

where, $I_1 = \frac{5}{6} \int \frac{2+6x}{1+2x+3x^2} dx$

Put $1 + 2x + 3x^2 = t \Rightarrow (2 + 6x)dx = dt$

$\therefore \quad I_1 = \frac{5}{6} \int \frac{dt}{t} = \frac{5}{6}\log|t| + C_1$

$= \frac{5}{6}\log\left|1 + 2x + 3x^2\right| + C_1$ $[t = 1 + 2x + 3x^2]$

where, $I_2 = \frac{11}{3} \int \frac{dx}{3x^2+2x+1}$

$= \frac{11}{9} \int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{3}\right]}$

$= \frac{11}{9} \int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{3}+\frac{1}{9}-\frac{1}{9}\right]}$

$= \frac{11}{9}\int \frac{dx}{\left[x^2+\frac{2x}{3}+\frac{1}{9}+\frac{1}{3}-\frac{1}{9}\right]}$

$= \frac{11}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}[\because (a+b)^2 = a^2+b^2+2ab]$

$= \frac{11}{9}\cdot\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)+C_2\left[\because \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$= \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C_2$

Putting the values of $I_1\ and\ I_2$ in Equation (ii),

$\Rightarrow I = \frac{5}{6}\log|1+2x+3x^2|+C_1-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)-C_2$

$\Rightarrow I = \frac{5}{6}\log|1+2x+3x^2|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C[\because C = C_1-C_2]$

27. Let P(A) = Probability that A solves the problem

P(B) = Probability that B solves the problem

$P(\overline{A})$= Probability that A does not solve the problem

and $P(\overline{B})$= Probability that B does not solve the problem

According to the question, we have

$P(A) = \frac{1}{2}$

then $P(\overline{A}) = 1-P(A) = 1-\frac{1}{2} = \frac{1}{2}$

$[\because\quad P(A)+P(\overline{A}) = 1]$

and $P(B) = \frac{1}{3}$

then $P(\overline{B}) = 1-P(B) = 1-\frac{1}{3} = \frac{2}{3}$

i. P (problem is solved)

$= P(A\cap\overline{B})+P(\overline{A}\cap B)+P(A\cap B)$

$= P(A)P(\overline{B})+P(\overline{A})\cdot P(B)+P(A)\cdot P(B)$

[∵ A and Bare independent events]

$= \left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right)$

$= \frac{2}{6}+\frac{1}{6}+\frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

Hence, probability that the problem is solved, is $\frac{2}{3}$

ii. P (exactly one of them solve the problem)

= P (A solve but B do not solve) + P (A do not solve but B solve)

$= P(A\cap\overline{B})+P(\ \overline{A}\cap B)$

$= P(A)\quad P(\overline{B})+P(\overline{A})P(B)$

$= \left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{1}{3}\right) = \frac{2}{6}+\frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

28. We have, to write the above integral as,

$I = \int_0^{2a} f(x)dx = \int_0^a f(x)dx+\int_a^{2a} f(x)dx$

$I = \int_0^a f(x)dx+I_1$

For l_1, Let 2a - t = x then dx = -dt

t = a, x = a

t = 2a, x = 0

$I_1 = \int_0^{2a} f(x) = \int_a^0 f(2a-t)(-dt)$

$= -\int_a^0 f(2a-t)dt$

$I_1 = \int_0^a f(2a-t)dt = \int_0^a f(2a-x)dx$

$I = \int_0^a f(x)dx+\int_0^a f(2a-x)dx$

$I = \int_0^a f(x)dx-\int_0^a f(x)$ ...[∵ f(2a - x) = -f(x)]

I = 0

Hence,

$\int_0^{2a} f(x)dx = 0$

OR

We can write Sin 2x = 2sinx.cosx

$\int e^{\sin x}\sin 2x\,dx = 2\int e^{\sin x}.\sin x\cos x\,dx$

Let Sin x = t

Cos xdx = dt

2 $\int$ $e^{\sin x}$ sin x cos xdx = 2 $\int e^t$ t dt

Using BY PARTS METHOD.

$2\int \mathrm{e}^{\mathrm{t}} \cdot \mathrm{tdt} = 2\left[\mathrm{t}\int \mathrm{e}^{\mathrm{t}}\mathrm{dt} - \int\left(\frac{\mathrm{dt}}{\mathrm{dt}} \cdot \int \mathrm{e}^{\mathrm{t}}\mathrm{dt}\right)\mathrm{dt}\right]$

$= 2\left[\mathrm{t} \cdot \mathrm{e}^{\mathrm{t}} - \int 1.\mathrm{e}^{\mathrm{t}}\mathrm{dt}\right]$

$= 2\left[\mathrm{t} \cdot \mathrm{e}^{\mathrm{t}} - \mathrm{e}^{\mathrm{t}}\right] + \mathrm{c}$

$= 2e^t(t-1) + c$

Replacing t with sin x

= $2e^{\sin x}$(sinx - 1) + c

29. $x\frac{dy}{dx} + y + \frac{1}{1+x^2} = 0$

$x\frac{dy}{dx} + y = -\frac{1}{1+x^2}$

$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = -\frac{1}{x(1+x^2)}$

So it a linear differential equation

P = $\frac{1}{x}$ and Q = $-\frac{1}{x(1+x^2)}$

$IF = e^{\int pdx} = e^{\int \frac{1}{x}dx} = e^{\log x} = \ x$

General solution

$y.IF = \int Q.IFdx + C$

$y.x = -\int \frac{1}{x(1+x^2)}.x\ dx + C$

$y.x = -\int \frac{1}{(1+x^2)}\ dx + C$

$y.x = -\tan^{-1}x + C$ ...(i)

At x =1, y = 0

$0 = -\tan^{-1}(1) + C$

$C = \frac{\pi}{4}$

Putting in eqn (i)

$y.x = -\tan^{-1}x + \frac{\pi}{4}$

OR

The given differential equation is,

$\left(1-x^2\right)\frac{dy}{dx} + xy = ax$

$\Rightarrow \frac{dy}{dx} + \frac{x}{1-x^2} \cdot y = \frac{ax}{1-x^2}$

This is of the form $\frac{dy}{dx} + Py = Q$

where $P = \frac{x}{1-x^2}$ and $Q = \frac{ax}{1-x^2}$

This the given differential equation is linear

Now, $IF = e^{\int Pdx}$

$= e^{\int \frac{xdx}{1-x^2}}$

$= e^{\frac{-1}{2}\int \frac{-2x}{1-x^2}dx} = e^{-\frac{1}{2}\log\left(1-x^2\right)} = \left(1-x^2\right)^{\frac{-1}{2}} = \frac{1}{\sqrt{1-x^2}}$

Solution is

$(IF) \cdot y = \int (IF)Q\ dx + C$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} \cdot y = \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{ax}{1-x^2}dx + C$

$\Rightarrow \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{\left(1-x^2\right)^{3/2}}dx + C$

$\Rightarrow \frac{y}{\sqrt{1-x^2}} = -\frac{a}{2}\int \frac{-2x}{\left(1-x^2\right)^{3/2}}dx + C$ ....(i)

Now, $I = \int \frac{-2x}{\left(1-x^2\right)^{3/2}}dx$

Let t = 1 - x^2

$\Rightarrow$ dt = -2x dx

$\therefore I = \int \frac{dt}{t^{3/2}} \Rightarrow I = \int t^{-3/2}dt$

$$\Rightarrow I = \frac{t^{\frac{-1}{2}}}{-\frac{1}{2}} \Rightarrow I = -2 \cdot \frac{1}{\sqrt{t}} = \frac{-2}{\sqrt{1-x^2}}$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-a}{2} \times \frac{-2}{\sqrt{1-x^2}} + C$$

$$\Rightarrow y = a + C\sqrt{1-x^2}$$

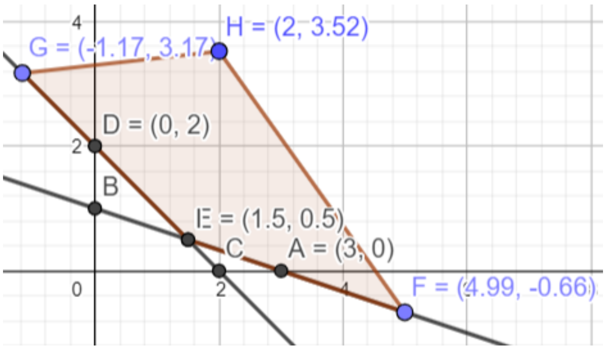
30. First, we will convert the given inequations into equations, we obtain the following equations:

$x_1 + 3x_2 = 3$, $x_1 + x_2 = 2$, $x_1 = 0$ and $x_2 = 0$

Region represented by $x_1 + 3x_2 \geq 3$:

The line $x_1 + 3x_2 = 3$ meets the coordinate axes at A(3,0) and B(0,1) respectively. By joining these points we obtain the line $x_1 + 3x_2 = 3$

Clearly (0,0) does not satisfies the inequation $x_1 + 3x_2 \geq 3$. So, the region in the plane which does not contain the origin represents the solution set of the inequation $x_1 + 3x_2 \geq 3$ Region represented by $x_1 + x_2 \geq 2$ The line $x_1 + x_2 = 2$ meets the coordinate axes at C(2,0) and D(0,2) respectively. By joining these points we obtain the line $x_1 + x_2 = 2$ Clearly (0,0) does not satisfies the inequation $x_1 + x_2 \geq 2$. So, the region containing the origin represents the solution set of the inequation $x_1 + x_2 \geq 2$ Region represented by $x_1 \geq 0$ and $x_2 \geq 0$ since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq 0$ and $x_2 \geq 0$ The feasible region determined by subject to the constraints are, $x_1 + 3x_2 \geq 3$, $x_1 + x_2 \geq 2$,and the non-negative restrictions $x_1 \geq 0$, and $x_2 \geq 0$, are as follows



The corner points of the feasible region are O(0,0), B(0,1), $E\left(\frac{3}{2}, \frac{1}{2}\right)$ and C(2,0)

The values of objective function at the corner points are as follows:

Corner point : $z = 3x_1 + 5x_2$

O(0, 0) : $3 \times 0 + 5 \times 0 = 0$

B(0, 1) : $3 \times 0 + 5 \times 1 = 5$

$E\left(\frac{3}{2}, \frac{1}{2}\right)$: $\frac{3}{2} + 5 \times \frac{1}{2} = 7$

C(2, 0) : $3 \times 2 + 5 \times 0 = 6$

Therefore, the minimum value of objective function Z is 0 at the point O(0,0). Hence, $x_1 = 0$ and $x_2 = 0$ is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 0.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

4x + y = 20, 2x + 3y = 30,x = 0 and y = 0

Region represented by 4x + y $\geq$ 20 :

The line 4x + y = 20 meets the coordinate axes at A(5,0) and B(0,20) respectively. By joining these points we obtain the line 4x + y = 20

Clearly (0,0) does not satisfies the inequation 4x + y $\geq$ 20 .

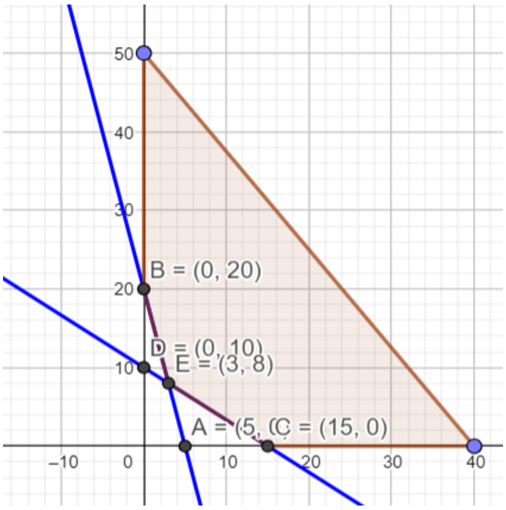
So, the region in xy plane which does not contain the origin represents the solution set of the inequation 4x + y $\geq$ 20

Region represented by 2x + 3y $\geq$ 30 :

The line 2x + 3y=30 meets the coordinate axes at C(15,0) and D(0,10) respectively. By joining these points we obtain the line 2x + 3y=30 .

Clearly (0,0) does not satisfies the inequation 2x + 3y $\geq$ 30 . So, the origin does not contain represents the solution set of the inequation 2x + 3y $\geq$ 30.

Region represented by x $\geq$ 0 and y $\geq$ 0 : graph will be in first quadrant since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the in equations x $\geq$ 0, and y $\geq$ 0 The feasible region determined by the system of constraints, 4x + y $\geq$ 20,2x + 3y $\geq$ 30,x $\geq$ 0, and y $\geq$ 0, are as follows.



The corner points of the feasible region are B(0,20), C(15,0), E(3,8) and C(15,0)

The values of Z at these corner points are as follows.

The value of objective function at the corner point : $Z = 18x + 10y$

B(0, 20) : $18 \times 0 + 10 \times 20 = 200$

E(3, 8) : $18 \times 3 + 10 \times 8 = 134$

C(15, 0) : $18 \times 15 + 10 \times 0 = 270$

Therefore, the minimum value of Z is 134 at the point E(3,8) . Hence, x = 3 and y = 8 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is 134.

31. Given function is

f(x) = $\begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$ = $\begin{cases} -x + 3, x \leq -3 \\ -2x, -3 < x < 3 \\ 6x + 2, x \geq 3 \end{cases}$

First, we verify continuity at x = - 3 and then at x = 3

Continuity at x = -3

LHL = $\lim_{x \to (-3)^-} f(x) = \lim_{x \to (-3)^-} (-x + 3)$

$\Rightarrow \quad \text{LHL} = \lim_{h \to 0} [-(-3 - h) + 3]$

$= \lim_{h \to 0} (3 + h + 3)$

= 3 + 3 = 6

and RHL = $\lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^+} (-2x)$

$\Rightarrow \quad RHL = \lim_{h \to 0} [-2(-3 + h)]$

$= \lim_{h \to 0} (6 - 2h)$

$\Rightarrow$ RHL = 6

Also, f(-3) = value of f(x) at x = - 3

=-(-3) + 3

= 3 + 3 = 6

$\because$ LHL = RHL = f(-3)

$\therefore$f(x) is continuous at x = - 3 So, x = - 3 is the point of continuity.

Continuity at x = 3

LHL = $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} [-(2x)]$

$\Rightarrow \quad LHL = \lim_{h \to 0} [-2(3 - h)]$

$= \lim_{h \to 0} (-6 + 2h)$

$\Rightarrow$ LHL = -6

and RHL = $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x + 2)$

$\Rightarrow \quad \mathrm{RHL} = \lim_{h \to 0}[6(3+h)+2]$

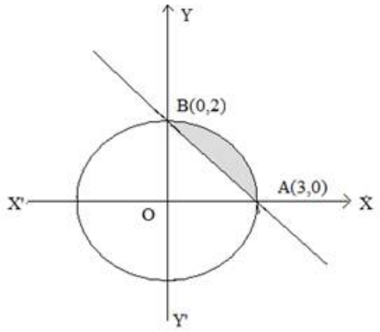
$\Rightarrow$ RHL = 20

$\because \quad \mathrm{LHL} \neq \mathrm{RHL}$

$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < -3 , -3< x < 3 and x > 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f(x).

Section D

32. 

$\frac{x^2}{9}+\frac{y^2}{4}=1$ ....(1)

$\frac{x}{3}+\frac{y}{2}=1$ ....(2)

$\Rightarrow \frac{x^2}{(3)^2}+\frac{y^2}{(2)^2}=1$ is the equation of ellipse and $\frac{x}{3}+\frac{y}{2}=1$ is the equation of intercept form of line.

On solving (1) and (2), we get points of intersection as (0,2) and (3,0).

Area $=\frac{2}{3}\int_0^3 \sqrt{9-x^2}dx-\int_0^3\left(\frac{6-2x}{3}\right)dx$

$=\frac{2}{3}[\frac{x}{2}\sqrt{3^2-x^2}+\frac{3^2}{2}sin^{-1}\frac{x}{3}]_0^3-\frac{1}{3}[6x-\frac{2x^2}{2}]_0^3$

{tex}=\frac{2}{3}[\frac{9π}{4}]-\frac{1}{3}[9]{/tex}

{tex}=\frac{3π}{2}-3{/tex}

$=\frac{3}{2}(\pi-2)$ sq unit.

33. Given that A = {1, 2, 3, 4},

a. Let R_1 = {(1,1), (1,2), (2,3), (2,2), (1,3), (3,3)}

   R_1 is reflexive, since, (1,1) (2,2) (3,3) lie is R_1

   Now, (1,2) $\in R_1$ (2,3) $\in R_1 \Rightarrow$ (1, 3) $\in R_1$

   Hence, R_1 is also transitive but (1,2), $\in R_1 \Rightarrow$ (2,1) $\notin R_1$

   So, it is not symmetric.

b. Let R_2 = {(1,2),(2,1)}. Here,1,2,3 $\in$ {1,2,3} but (1,1),(2,2),(3,3) are not in R.

   Therefore, R is not reflexive.Now, (1,2) $\in R_2$, (2,1) $\in R_2$

   So, it is symmetric.

   Now (1,2) $\in$ R (2,1) $\in$ R, but (1,1) $\notin$ R,

   therefore, R is not transitive.

c. Let R3 = {(1,2), (2,1),(1,1)(2,2),(3,3),(1,3),(3,1),(2,3)}

   Clearly, R_3 is reflexive, symmetric and transitive.

OR

Given that A = [-1, 1]

i. $f(x)=\frac{x}{2}$

   Let $f(x_1) = f(x_2)$

   $\Rightarrow \frac{x_1}{2}=\frac{x_2}{2} \Rightarrow$ $x_1 = x_2$

   So, f(x) is one-one.

   Now, let $y=\frac{x}{2}$

   $\Rightarrow x=2y \notin A,\ \forall y \in A$

   As for $y=1 \in A,\ x=2 \notin A$

   So, f(x) is not onto.

   Also, f(x) is not bijective as it is not onto.

ii. g(x) = |x|

Let g(x_1) = g(x_2)

$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$

So, g(x) is not one-one.

Now, x=±y $\notin A$ for all y ∈ R

So, g(x) is not onto, also, g(x) is not bijective.

iii. h(x) = x|x|

$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$

So, h(x) is one-one

Now, let y = x|x|

$\Rightarrow y = x^2 \in A, \forall x \in A$

So, h(x) is onto also, h(x) is a bijective.

iv. k(x) = x^2

Let k(x_1) = k(x_2)

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, k(x) is not one-one.

Now, let y = x^2

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A\ x = \sqrt{y} \notin A, \forall y \in A$

As for y = -1, $x = \sqrt{-1} \notin A$

Hence, k(x) is neither one-one nor onto.

34. Put $\frac{1}{x} = u, \frac{1}{y} = v$ and $\frac{1}{z} = w$ in the given equations,

$2u + 3v + 10w = 4;\ 4u - 6v + 5w = 1;\ 6u + 9v - 20w = 2$

$\therefore$ The matrix form of given equations is $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} x \\ v \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ [AX= B]

Here, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X \begin{bmatrix} x \\ v \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$

$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$

$= 150 + 330 + 750 = 1200 \neq 0$

$\therefore A^{-1}$exists and unique solution is $X = A^{-1}B \ldots$(i)

Now $A_{11} = 75, A_{12} = 110, A_{13} = 72$ and $A_{21} = 150, A_{22} = -100, A_{23} = 0$ and $A_{31} = 75, A_{32} = 30, A_{33} = -24$

$\therefore adj.\ A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

And $A^{-1} = \frac{adj.A}{|A|} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$\therefore$ From eq. (i),

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

$\therefore u = \frac{1}{2}, v = \frac{1}{3}, w = \frac{1}{5}$

$\Rightarrow x = \frac{1}{u} = 2, y = \frac{1}{v} = 3, z = \frac{1}{w} = 5$

35. Here the equation of two planes are: $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Since the line is parallel to the two planes.

$\therefore$ Direction of line $\overrightarrow{b}$ = $(\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$

= $-3\hat{i} + 5\hat{j} + 4\hat{k}$

$\therefore$ Equation of required line is

$\vec{r}$= $(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$ .... (i)

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

For this line to intersect the plane $\overrightarrow{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$ we have

$(1 - 3\lambda)2$ + $(2 + 5\lambda)1$ + $(3 + 4\lambda)1$ = 4

$\Rightarrow \lambda$= 1

$\therefore$ Point of intersection is (4, -3, -1)

OR

Here,it is given that

$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$

Here,we have

$\overrightarrow{a_1} = 6\hat{i} + 3\hat{k}$

$\overrightarrow{\mathrm{b}_1} = 2\hat{\mathrm{i}} - \hat{\mathrm{j}} + 4\hat{\mathrm{k}}$

$\overrightarrow{a_2} = -9i + j - 10\hat{k}$

$\overrightarrow{\mathrm{b}_2} = 4\hat{i} + \hat{j} + 6\hat{\mathrm{k}}$

Thus,

$\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{\mathrm{k}} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$

$= i(-6 - 4) - j(12 - 16) + \hat{\mathrm{k}}(2 + 4)$

$\therefore \overrightarrow{b_1} \times \overrightarrow{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore |\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}| = \sqrt{(-10)^2 + 4^2 + 6^2}$

$= \sqrt{100 + 16 + 36}$

$= \sqrt{152}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (-9 - 6)\hat{\mathrm{i}} + (1 - 0)\hat{j} + (6 - 3)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$

Now,

$(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}) \cdot (\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1}) = (-10\hat{i} + 4\hat{j} + 6\hat{\mathrm{k}}) \cdot (-15\hat{i} + \hat{j} + 3\hat{k})$

$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$

= 150 + 4 + 18

= 172

Thus, the shortest distance the given lines is

$\mathrm{d} = \left| \frac{(\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}) \cdot (\overrightarrow{\mathrm{a}_2} - \overrightarrow{\mathrm{a}_1})}{|\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2}|} \right|$

$\therefore \mathrm{d} = \left| \frac{172}{\sqrt{152}} \right|$

$\therefore \mathrm{d} = \frac{172}{2\sqrt{38}}$

$\therefore \mathrm{d} = \frac{86}{\sqrt{38}}$

$d = \frac{86}{\sqrt{38}}$ units

Section E

36. i. x + 0.21 = 0.44 ⇒ x = 0.23

ii. 0.41 + y + 0.44 + 0.11 = 1 ⇒ y = 0.04

iii. $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$

P(B) = 0.09 + 0.04 + 0.23 = 0.36

$P\left(\frac{C}{B}\right) = \frac{0.23}{0.36} = \frac{23}{36}$

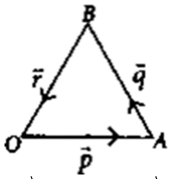
OR

P(A or B but not C)

= 0.32 + 0.09 + 0.04

= 0.45

37. i. Let OAB be a triangle such that



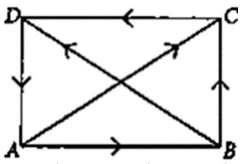
$\overrightarrow{AO} = -\vec{p}, \overrightarrow{AB} = \vec{q}, \overrightarrow{BO} = \vec{r}$

Now, $\vec{q} + \vec{r} = \overrightarrow{AB} + \overrightarrow{BO}$

$= \overrightarrow{AO} = -\vec{p}$

ii. From triangle law of vector addition,

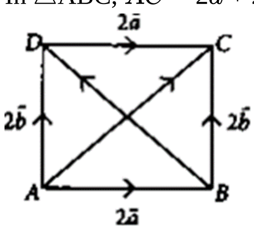
$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} + \overrightarrow{CD}$



= $\overrightarrow{AB} + 2\overrightarrow{BC} + \overrightarrow{CD}$

= $\overrightarrow{AB} + 2\overrightarrow{BC} - \overrightarrow{AB} = 2\overrightarrow{BC}$ $[\because \overrightarrow{AB} = -\overrightarrow{CD}]$

iii. In △ABC, $\overrightarrow{AC}$ = $2\vec{a}$ + $2\vec{b}$ ...(i)



and in △ABD, $2\vec{b}$ = $2\vec{a}$ + $\overrightarrow{BD}$ ...(ii) [By triangle law of addition]

Adding (i) and (ii), we have $\overrightarrow{AC} + 2\vec{b} = 4\vec{a} + \overrightarrow{BD} + 2\vec{b}$

$\Rightarrow \overrightarrow{AC} - \overrightarrow{BD} = 4\vec{a}$

OR

Since T is the mid point of YZ

So, $\overrightarrow{YT} = \overrightarrow{TZ}$

Now, $\overrightarrow{XY} + \overrightarrow{XZ} = (\overrightarrow{XT} + \overrightarrow{TY}) + (\overrightarrow{XT} + \overrightarrow{TZ})$ [By triangle law]

= $2\overrightarrow{XT} + \overrightarrow{TY} + \overrightarrow{TZ} = 2\overrightarrow{XT}$ $[\because \overrightarrow{TY} = -\overrightarrow{YT}$]

38. i. We have, I(x) = $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$

⇒ I'(x) = $\frac{-2000}{x^3} + \frac{250}{(600-x)^3}$ and

⇒ I''(x) = $\frac{6000}{x^4} + \frac{750}{(600-x)^4}$

For maxima/minima, I'(x) - 0

$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3} \Rightarrow 8(600 - x)^3 = x^3$

Taking cube root on both sides, we get

2(600 - x) = x ⇒ 1200 = 3x ⇒ x = 400

Thus, I(x) is minimum when you are at 400 feet from the strong intensity lamp post.

ii. At a distance of 200 feet from the weaker lamp post.

Since I(x) is minimum when x = 400 feet, therefore the darkest spot between the two light is at a distance of 400 feet from a stronger lamp post, i.e., at a distance of 600 - 400 = 200 feet from the weaker lamp post.

iii. $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be 600 - x.

So, the combined light intensity from both lamp posts is given by $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$.

OR

We know that l(x) = $\frac{1000}{x^2}+\frac{125}{(600-x)^2}$

When x = 400

l(x) = $\frac{1000}{160000}+\frac{125}{(600-400)^2}$

= $\frac{1}{160}+\frac{125}{40000}=\frac{1}{160}+\frac{1}{320}=\frac{3}{320}$ units