Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. The number of all possible matrices of order 3×3 with each entry 0 or 1 is **[1]**

a) 18 b) 27

c) 512 d) 81

2. For non-singular square matrices A and B of the same order, we have adj (AB) = ? **[1]**

a) (adj B) (adj A) b) |AB|

c) (adj B).|A| d) (adj A).|B|

3. If $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$, then the cofactor A_{21} is **[1]**

a) hc - fg b) -(hc + fg)

c) fg - hc d) fg + hc

4. Which of the following statements is true for the function f(x) = $\begin{cases} x^2+3, & x \neq 0 \\ 1, & x = 0 \end{cases}$? **[1]**

a) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$ b) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$ - {0}

c) f(x) is discontinuous at infinitely many d) f(x) is continuous $\forall x \in \mathbb{R}$

points

5. The angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ is **[1]**

a) $\cos^{-1}\left(\frac{3}{4}\right)$ b) $\cos^{-1}\left(\frac{2}{3}\right)$

c) $\frac{\pi}{3}$ d) $\cos^{-1}\left(\frac{5}{6}\right)$

6. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are: **[1]**

a) 2, degree not defined b) 2, 2

c) 2, 3 d) 1, 3

7. Maximize Z = 50x + 60y , subject to constraints x +2 y ≤ 50 , x +y ≥ 30, x, y ≥ 0. **[1]**

a) 2500 b) 1600

c) 1547 d) 1525

8. The 2 vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a $\triangle$ABC. The length of the median through A is **[1]**

a) $\frac{\sqrt{48}}{2}$ b) $\frac{\sqrt{34}}{2}$

c) $\sqrt{18}$ d) $\sqrt{22}$

9. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx=$? **[1]**

a) 2 b) -1

c) 1 d) 0

10. If A and B are matrices of same order, then (AB′ – BA′) is a **[1]**

a) skew-symmetric matrix b) unit matrix

c) symmetric matrix d) null matrix

11. The linear programming problem minimize Z = 3x + 2y subject to constraints x + y $\geq$ 8, 3x + 5y $\leq$15, x $\geq$ 0 and y $\geq$ 0, has **[1]**

a) one solution b) infinitely many solutions

c) no feasible solution d) two solutions

12. If $\vec{a}=(2\hat{i}+4\hat{j}-\hat{k})$ and $\vec{b}=(3\hat{i}-2\hat{j}+\lambda\hat{k})$ be such that $\vec{a}\perp\vec{b}$ then $\lambda=$? **[1]**

a) 3 b) -3

c) -2 d) 2

13. If $A=\begin{bmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 3 & -4\end{bmatrix}$ is singular then k =? **[1]**

a) $\frac{16}{3}$ b) $\frac{33}{2}$

c) $\frac{34}{5}$ d) $\frac{33}{4}$

14. If $\mathrm{P}(\mathrm{A}\mid\mathrm{B})=\mathrm{P}\left(\mathrm{A}'\mid\mathrm{B}\right)$, then which of the following statements is true? **[1]**

a) P(A) = 2 P(B) b) $P(A\cap B)=2P(B)$

c) $P(A \cap B) = \frac{1}{2}P(B)$

d) $\mathrm{P(A)} = \mathrm{P}\left(\mathrm{A}'\right)$

15. A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution **[1]**

a) $y = \ \nu x$

b) $\nu = \ yx$

c) $x = \ \nu$

d) $x = \ \nu y$

16. If $|\overrightarrow{\mathrm{a}}|$ = 3, $|\overrightarrow{\mathrm{b}}|$ = 4 and $|\vec{a} \times \vec{b}|$ = 6, then the value of $\vec{a} \cdot \vec{b}$ is **[1]**

a) $6\sqrt{3}$

b) $3\sqrt{3}$

c) 6

d) 12

17. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then $\frac{dy}{dx}$ = ? **[1]**

a) $\frac{-\sqrt{x}}{\sqrt{y}}$

b) $\frac{1}{2}\frac{-\sqrt{y}}{\sqrt{x}}$

c) $\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$

d) $\frac{-\sqrt{y}}{\sqrt{x}}$

18. The direction ratios of two lines are a, b, c and (b - c), (c - a), (a - b) respectively. The angle between these lines is **[1]**

a) $\frac{\pi}{3}$

b) $\frac{\pi}{4}$

c) $\frac{3\pi}{4}$

d) $\frac{\pi}{2}$

19. **Assertion (A):** f(x) = tan x - x always increases. **[1]**

Reason (R): Any function y = f(x) is increasing if $\frac{dy}{dx}$ > 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The Relation R given by R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1) on set A = {1, 2, 3, 2} is symmetric. **[1]**

Reason (R): For symmetric Relation R = R^{-1}

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$ **[2]**

OR

Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$.

22. Find the maximum and minimum values of $f(x) = \left(\sin x + \frac{1}{2}\cos x\right)$ in $0 \leq x \leq \frac{\pi}{2}$ **[2]**

23. Find the intervals of function f(x) = 6 + 12x + $3x^2$ - $2x^3$ is **[2]**

a. increasing
b. decreasing.

OR

If y = 7x - x^3 and x increases at the rate of 4 units per second, how fast is the slope of the curve changing when x = 2?

24. Evaluate: $\int_0^a \frac{x^4}{\sqrt{a^2 - x^2}}dx$ **[2]**

25. Find the local maxima and local minima, of function. Find also the local maximum and the local minimum value, as the case may be: f(x) = (x - 1) (x + 2)^2 **[2]**

Section C

26. $\int \frac{x^2\left(x^4+4\right)}{x^2+4} dx$ **[3]**

27. A bag contains 7 red, 5 white and 8 black balls. If four balls are drawn one by one with replacement, what is the probability that all are white? **[3]**

28. Find $\int \frac{\cos \theta}{\left(4+\sin^2 \theta\right)\left(5-4 \cos^2 \theta\right)} d\theta.$ **[3]**

OR

Evaluate: $\int_0^{\pi/6}\left(2+3x^2\right) \cos 3x dx$

29. Solve the differential equation: (y^2 - 2xy) dx = (x^2 - 2xy) dy **[3]**

OR

Solve the initial value problem: xe$^{y/x}$ - y sin $\left(\frac{y}{x}\right)$ + $x\frac{dy}{dx}$ sin $\left(\frac{y}{x}\right)$ = 0, y(1) = 0

30. Solve the Linear Programming Problem graphically: **[3]**

Maximize Z = 50x + 30y Subject to

2x + y $\leq$ 18

3x + 2y $\leq$ 34

x, y $\geq$ 0

OR

Solve the following LPP graphically:

Minimize Z = 3x + 5y

Subject to

- 2x + y $\leq$ 4

x + y $\geq$ 3

x - 2 y $\leq$ 2

x, y $\geq$ 0

31. If x = a$\left(\cos t+\log \tan \frac{t}{2}\right)$, y = a sin t, then evaluate $\frac{d^2 y}{dx^2}$ at $t=\frac{\pi}{3}$. **[3]**

Section D

32. Find the area of the region enclosed by the parabola y^2 = x and the line x + y = 2. **[5]**

33. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a - b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}. **[5]**

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A ⇒B defined by$f(x)=\left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. Given $A=\begin{bmatrix}2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2\end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17. **[5]**

35. Find the shortest distance $\overrightarrow{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\overrightarrow{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$. **[5]**

OR

Find the image of the point with position vector $3\hat{i}+\hat{j}+2\hat{k}$ in the plan $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=4$ Also, find the position vectors of the foot of the perpendicular and the equation of the perpendicular line through $3\hat{i}+\hat{j}+2\hat{k}$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

i. What is the probability that the shell fired from exactly one of them hit the plane? (1)

ii. If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B? (1)

iii. What is the probability that the shell was fired from A? (2)

OR

How many hypotheses are possible before the trial, with the guns operating independently? Write the conditions of these hypotheses. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

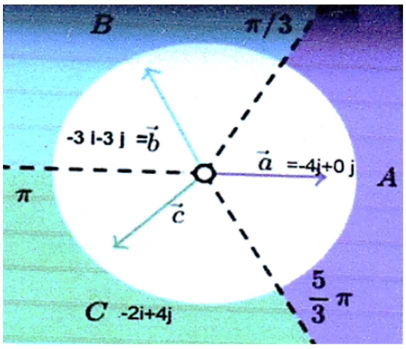
Three friends Ganesh, Dinesh and Ramesh went for playing a Tug of war game. Team A, B, and C belong to Ganesh, Dinesh and Ramesh respectively.

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team B → $F_2 = -2\hat{i} + 4\hat{j}$ KN

Team C → $F_3 = -3\hat{i} - 3\hat{j}$ KN



i. Which team will win the game? (1)

ii. What is the magnitude of the teams combine Force? (1)

iii. What is the magnitude of the force of Team B? (2)

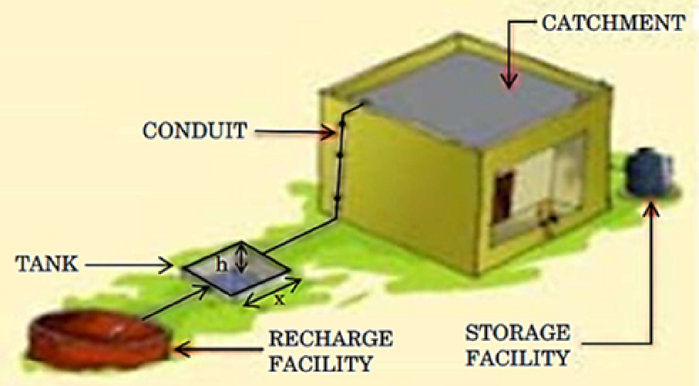
OR

How many KN Force is applied by Team A? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



i. Find the total cost C of digging the tank in terms of x. (1)

ii. Find $\frac{dC}{dx}$. (1)

iii. Find the value of x for which cost C is minimum. (2)

OR

Check whether the cost function C(x) expressed in terms of x is increasing or not, where x > 0. (2)

Solution

Section A

1.

(c) 512

Explanation:

As order of 3 × 3 matrix contains 9 elements. Each element can be selected in 2 ways (it can be either 0 or 1).

Hence, all the nine netries can be chosen in 2^9 = 512 ways (by the multiplication principle)

2. **(a)** (adj B) (adj A)

Explanation:

We Know that $(AB)^{-1} = \frac{adj(AB)}{|AB|}$

adj (AB) = $(AB)^{-1}$.|AB|

We also know that $(AB)^{-1} = B^{-1}\ A^{-1}$

|AB| = |A|.|B|

Putting them in (i)

adj (AB) = $B^{-1}A^{-1}$.|A|.|B|

adj (AB) = $(B^{-1}$,|B|)$(A^{-1}$, |A|)

adj (AB) = (adjB).(adjA)

3. **(a)** hc - fg

Explanation:

$$A_{21} = (-1)^{2+1}M_{21} = -M_{21}\ = -\begin{vmatrix} h & g \\ f & c \end{vmatrix}$$

= -(hc - fg) = fg - hc

4.

(b) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$ - {0}

Explanation:

f(x) is continuous and differentiable $\forall x \in \mathbb{R}$ - {0}

5.

(b) $\cos^{-1}\left(\frac{2}{3}\right)$

Explanation:

Direction ratios of the lines are given $2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + \hat{\jmath} + 8\hat{k}$

$\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{2^2 + 2^2 + 1}$

$|\vec{a}| = 3$

$\overrightarrow{\mathrm{b}} = 4\hat{\mathrm{i}} + \hat{\mathrm{j}} + 8\hat{\mathrm{k}}$

$|\overrightarrow{\mathrm{b}}| = \sqrt{4^2 + 1 + 8^2}$

= 9

$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$

$\cos\alpha = \frac{(2\hat{\imath}+2\hat{\jmath}+\hat{\mathrm{k}})\cdot(4\hat{\mathrm{i}}+\hat{\mathrm{j}}+8\hat{\mathrm{k}})}{3\times9}$

$\cos\alpha = \frac{8+8+2}{27}$

$\cos\alpha = \frac{2}{3}$

6. **(a)** 2, degree not defined

   Explanation:

   2, degree not defined

7. **(a)** 2500

   Explanation:

   Here , Maximize Z = 50x+60y , subject to constraints x +2 y ≤ 50 , x + y ≥ 30, x, y ≥ 0.

Corner points	Z = 50x +60 y
P(50 ,0)	2500
Q(0 , 30)	1800
R(10, 20)	1700

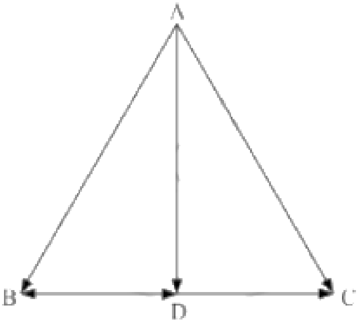
   Hence, the maximum value is 2500

8.

   (b) $\frac{\sqrt{34}}{2}$

   Explanation:

   In $\triangle ABC$,



   Using the triangle law of vector addition, we have

   $\overrightarrow{BC}=\overrightarrow{AC}-\overrightarrow{AB}$

   $=(3\hat{i}-\hat{j}+4\hat{k})-(\hat{j}+\hat{k})$

   $\therefore \overrightarrow{BD}=\frac{1}{2}\overrightarrow{BC}=\frac{3}{2}\hat{i}-\hat{j}+\frac{3}{2}\hat{k}$ (since AD is the median)

   In $\triangle \mathrm{ABD}$, using the triangle law of vector addition, we have

   $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{BD}$

   $=(\hat{j}+\hat{k})+\left(\frac{3}{2}\hat{i}-\hat{j}+\frac{3}{2}\hat{k}\right)$

   $=\frac{3}{2}\hat{i}+0\hat{j}+\frac{5}{2}\hat{k}$

   $\therefore AD=\sqrt{\left(\frac{3}{2}\right)^2+0^2+\left(\frac{5}{2}\right)^2}=\frac{1}{2}\sqrt{34}$

   Hence, the length of the median through A is $\frac{1}{2}\sqrt{34}$ units.

9. **(a)** 2

   Explanation:

   cos x is an even function so,

   $\int_{-a}^{a} f(x)dx=2\int_{0}^{a} f(x)dx$

   $\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx=2\int_{0}^{\frac{\pi}{2}} \cos x dx$

   = 2 (1 - 0)

   = 2

10. **(a)** skew-symmetric matrix

    Explanation:

    We have matrices A and B of same order.

    Let P = (AB' − BA')

    Then, P′ = (AB′ − BA′)'

= (AB′)′ − (BA′)′

= (B′)′ (A)′ − (A′)′B′ = BA′ − AB′ = −(AB′ − BA′) = −P

Therefore, the given matrix (AB - BA') is a skew-symmetric matrix.

11.

(c) no feasible solution

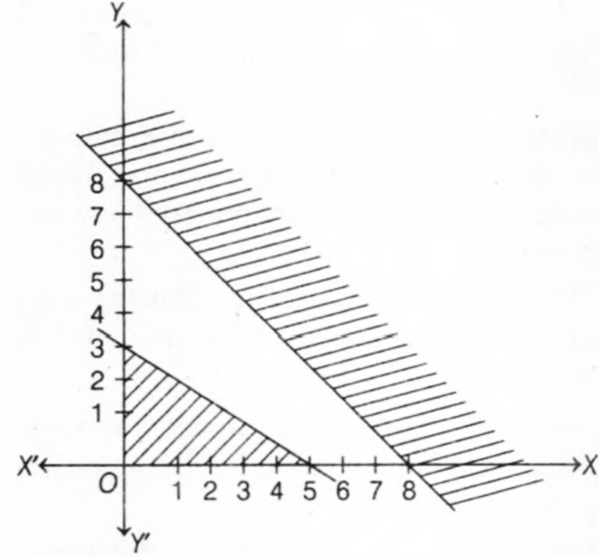
Explanation:

Table for equation x + y = 8 is

x	0	8
y = 8 - x	8	0

Table for equation 3x + 5y = 15 is

x	0	5
y = $\frac{15-3x}{5}$	3	0



It can be concluded from the graph, that there is no point, which can satisfy all the constraints simultaneously. Therefore, the problem has no feasible solution.

12.

(c) -2

Explanation:

Given $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}\ and\ \vec{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ such that $\vec{a} \perp \vec{b}$

$\therefore$ 6 - 8 - $\lambda = 0$ = -2 - $\lambda = 0$

$\Rightarrow \lambda = -2$

13.

(b) $\frac{33}{2}$

Explanation:

When a given matrix is singular then the given matrix determinant is 0.

|A| = 0

Given,

$$A = \begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$$

|A| = 0

1(-4k + 6) -k(-12 + 4) + 3(9 - 2k) = 0

-4k + 6 + 12k - 4k + 27 - 6k = 0

-2k + 33 = 0

k $= \frac{33}{2}$.

Which is the required solution.

14\.

(c) $P(A \cap B) = \frac{1}{2} P(B)$

Explanation:

$P(A \cap B) = \frac{1}{2} P(B)$

15\.

(d) $x = \ \nu y$

Explanation:

A homogeneous equation of the form $\frac{dy}{dx} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution x= vy.so that it becomes variable separable form and integration is then possible

16\. **(a)** $6\sqrt{3}$

Explanation:

$|\vec{a}| = 3|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta.$

$6 = 3 \times 4 \sin\theta.$

$\frac{bx\cancel{2}}{3 \times 4_2} = \sin\theta.$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\vec{a} \cdot \bar{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$

$= 3 \times 4 \times \cos\frac{\pi}{6}$

$= 3 \times 4 \times \frac{\sqrt{3}}{2}$

$= 6\sqrt{3}.$

17\.

(d) $\frac{-\sqrt{y}}{\sqrt{x}}$

Explanation:

Given that $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating with respect to x, we obtain

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$

Or $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$

18\.

(d) $\frac{\pi}{2}$

Explanation:

Let's consider the first parallel vector to be $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be

$\overrightarrow{\mathrm{b}} = (\mathrm{b} - \mathrm{c})\hat{\mathrm{i}} + (\mathrm{c} - \mathrm{a})\hat{\mathrm{j}} + (\mathrm{a} - \mathrm{b})\hat{\mathrm{k}}$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\overrightarrow{\mathrm{b}}| = \sqrt{(\mathrm{b} - \mathrm{c})^2 + (\mathrm{c} - \mathrm{a})^2 + (\mathrm{a} - \mathrm{b})^2}$

$= \sqrt{2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)}$

$\Rightarrow \cos\alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot ((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{\sqrt{2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)} \times \sqrt{a^2 + b^2 + c^2}}$

$$\Rightarrow \cos\alpha = \frac{ab-ac+bc-ba+ca-cb}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$$
$$\Rightarrow \cos\alpha = \frac{0}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$$
$$\Rightarrow \alpha = \cos^{-1}(0)$$
$$\therefore \alpha = \frac{\pi}{2}$$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20.

(d) A is false but R is true.

Explanation:

R = {(1, 3), (4, 2) (2, 7) (2, 3) (3, 1)}

As (2, 3) ∈ R but (3, 2) $\notin$ R

So, set 'A' is not symmetric.

Section B

21. Let $\cot^{-1}\left(\frac{-5}{12}\right) = y$

Then $\cot y = \frac{-5}{12}$

Now,

$\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right] = \sin 2y$

$= 2\sin y\cos y = 2\left(\frac{12}{13}\right)\left(\frac{-5}{13}\right)$ $\left[\text{since } \cot y < 0, \text{ so } y \in \left(\frac{\pi}{2}, \pi\right)\right]$

$= \frac{-120}{169}$

OR

We have to find principle value of , $\cos^{-1}\left(\frac{1}{2}\right)$

Lets $\cos^{-1}\frac{1}{2} = \theta$

$\Rightarrow \frac{1}{2}$ = cos θ

$\Rightarrow$ cos $\frac{\pi}{3}$ = cos θ

$\Rightarrow \theta = \frac{\pi}{3}$

22. We have Max. value is $\frac{3}{4}$ at $x = \frac{\pi}{6}$ and min. value is $\frac{1}{2}$ at $x = \frac{\pi}{2}$

Also F'(x) = cos x - $\frac{1}{2}$sin x = 0

2 cos x = sin x

$\frac{\pi}{6} = \frac{\pi}{3}$

$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \frac{1}{2}\cos\frac{\pi}{2} = \frac{1}{2}$

$f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} + \frac{1}{2}\cos\frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{4}$

$f\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} + \frac{1}{2}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2} + \frac{1}{4}$

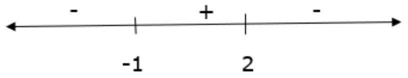
23. The given function is,

f (x) = -2x^3 + 3x^2 + 12x + 6

f’(x) = -6x^2 + 6x + 12

f’(x) = -6(x^2 - x - 2)

= -6(x - 2)(x + 1)



Function f(x) is increasing for x ∈ [-1, 2] and decreasing in x ∈ (-∞, -1) ∪ (2, ∞).

OR

Here,

$\frac{dx}{dt} = 4 \text{ units} / \sec$ and $x = 2$

And, $y = 7x - x^3$

Slope of the curve (S) $= \frac{dy}{dx}$

$S = \frac{dy}{dx} = 7 - 3x^2$

$\frac{ds}{dt} = -6x \times \frac{dx}{dt}$

$= -6(2)(4)$

= - 48 units/ sec

So , slope is decreasing at the rate of 48 units/ sec

24. For the given integral is

Let $x = a\sin\theta$. Then, $dx = d(a\sin\theta) = a\cos\theta d\theta$

Also, $x = 0 \Rightarrow a\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

And, $x = a \Rightarrow a\sin\theta = a \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\therefore \quad I = \int_0^a \frac{x^4}{\sqrt{a^2-x^2}}dx = \int_0^{\pi/2} \frac{(a\sin\theta)^4}{\sqrt{a^2-a^2\sin^2\theta}} a\cos\theta d\theta = a^4\int_0^{\pi/2}\sin^4\theta d\theta$

$\Rightarrow \quad I = a^4 \times \frac{3\pi}{16} = \frac{3\pi a^4}{16}$

25. Given: $f(x) = (x - 1)(x + 2)^2$

$\because$ f'(x) = (x + 2) + 2(x - 1) (x + 2)

= (x + 2) (x + 2 + 2x - 2)

= (x + 2) (3x)

To find the point of maxima and minima we must have

f'(x) = 0

$\Rightarrow$ (x + 2) × 3x = 0

$\Rightarrow$ x = 0, - 2

At x = -2 f'(x) changes from + ve to - ve

$\because$ x = -2 is point of local maxima

At x = 0 f'(x) changes from - ve to -ve

$\because$ x = 0 is point of local minima

Thus, local min value = f(0) = - 4

local max value f(-2) = 0.

Section C

26. Let I = $\int \frac{x^2(x^4+4)}{x^2+4}dx$

$= \int\left(\frac{x^6+4x^2}{x^2+4}\right)dx$

Therefore by long division we have

$$
\begin{array}{l}
\qquad\quad x^4-4x^2+20 \\
x^2+4\overline{)\,x^6+4x^2} \\
\qquad x^6+4x^4 \\
\qquad - \quad - \\
\hline
\qquad -4x^4+4x^2 \\
\qquad -4x^4-16x^2 \\
\qquad\ + \qquad + \\
\hline
\qquad\quad 20x^2 \\
\qquad\quad 20x^2 + 80 \\
\qquad\quad - \qquad - \\
\hline
\qquad\quad -80 \\
\hline
\end{array}
$$

Therefore,

$\frac{x^2(x^4+4)}{(x^2+4)} = (x^4 - 4x^2 + 20) - \frac{80}{x^2+4}$

I = $\int \frac{x^2(x^4+4)}{(x^2+4)}dx$

$= \int (x^4 - 4x^2 + 20)\,dx - 80\int\frac{dx}{x^2+2^2}$

$= \int x^4dx - 4\int x^2dx + 20\int dx - 80\int\frac{dx}{x^2+2^2}$

= $\frac{x^{4+1}}{4+1} - 4\left[\frac{x^3}{3}\right]$ + 20(x) - 80 $\times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

= $\frac{x^5}{5} - \frac{4}{3}x^3$ + 20x - 40 $\tan^{-1}$ $\left(\frac{x}{2}\right)$ + C

27. Let p be the probability of getting 1 white ball out of 7 red, 5 white and 8 black balls. Therefore,we have,

$p = \frac{5}{20}$

$q = 1 - \frac{1}{4}$

$p = \frac{1}{4}$ [since p + q = 1]

$q = \frac{3}{4}$

Let X be a random variable denoting the number of white balls selected with replacement out of 4 balls. Then,

Probability of getting r white balls out of n balls is given by

P(X=r) = ${}^nC_r p^r q^{n-r}$

Probability of getting all white balls

= P(X = 4)

$=^4 C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^{4-4}$

$= \left(\frac{1}{4}\right)^4$

28. According to the question, $I = \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)]}d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)[5-4(1-\sin^2\theta)]}d\theta$ $[\because cos^2\theta = 1 - sin^2\theta]$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(5-4+4\sin^2\theta)}d\theta$

$= \int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}d\theta$

Let $\sin\theta = t \Rightarrow \cos\theta d\theta = dt$

Then, $I = \int \frac{dt}{(4+t^2)(1+4t^2)}$

let , $\frac{1}{(4+t^2)(1+4t^2)} = \frac{A}{4+t^2} + \frac{B}{1+4t^2}$

using partial fractions

At t = 0, $\frac{A}{4} + \frac{B}{1} = \frac{1}{4\times1} \Rightarrow \quad A + 4B = 1$ ...(i)

At $t = 1$, $\frac{A}{5} + \frac{B}{5} = \frac{1}{5\times5} \Rightarrow 5A + 5B = 1$ ...(ii)

On solving Equations (i) and (ii), we get

$A = \frac{-1}{15}\ \ and\ \ B = \frac{4}{15}$

$\frac{1}{(4+t^2)(1+4t^2)} = \frac{-\frac{1}{15}}{4+t^2} + \frac{\frac{4}{15}}{1+4t^2}$

$\Rightarrow \quad \frac{1}{(4+t^2)(+4t^2)} = \frac{-1}{15(4+t^2)} + \frac{4}{15(1+4t^2)}$

Integrating both sides w.r.t. t,

$\Rightarrow \int \frac{1}{(4+t^2)(1+4t^2)}dt = \frac{-1}{15}\int \frac{1}{4+t^2}dt + \frac{4}{15}\int \frac{1}{1+4t^2}dt$

$= \frac{-1}{15}\int \frac{1}{2^2+t^2} + \frac{4}{15\times4}\int \frac{1}{\left(\frac{1}{2}\right)^2+t^2}dt$

$= \frac{-1}{15}\cdot\frac{1}{2}\tan^{-1}\frac{t}{2} + \frac{1}{15}\cdot\frac{1}{1/2}\tan^{-1}\frac{t}{1/2} + C \ \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

put $t = \sin\theta$

$= \frac{-1}{30}\tan^{-1}\frac{\sin\theta}{2} + \frac{2}{15}\tan^{-1}2\sin\theta + C$

OR

Let I = $\int_0^{\pi/6}\left(2 + 3x^2\right)\cos 3x dx$.

Then, solving by means of integration by parts, we have

$I = \left[\left(2 + 3x^2\right)\times\frac{1}{3}\sin 3x\right]_0^{\pi/6} - \int_0^{\pi/6} 6x \times \frac{1}{3}\sin 3x dx$

$= \left[\frac{1}{3}\left(2 + 3x^2\right)\sin 3x\right]_0^{\pi/6} - 2\int_0^{\pi/6} x\sin 3x dx$

$= \left[\frac{1}{3}\left(2 + 3x^2\right)\sin 3x\right]_0^{\pi/6} - 2\ \left[\left[\frac{-x\cos 3x}{3}\right]_0^{\pi/6} - \int_0^{\pi/6} - \frac{\cos 3x}{3}dx\right]$

$= \left[\frac{1}{3}\left(2 + 3x^2\right)\sin 3x\right]_0^{\pi/6} - 2\ \left[\left[\frac{-x\cos 3x}{3}\right]_c^{\pi/6} + \frac{1}{9}[\sin 3x]_0^{\pi/6}\right]$

$= \left[\frac{1}{3}\left(2 + \frac{\pi^2}{12}\right)\sin\frac{\pi}{2} - \frac{1}{3}(2 + 0)0\right] - 2\ \left[\left\{\left(-\frac{\pi}{18}\cos\frac{\pi}{2}\right) + \frac{0\cos 0}{3}\right\} + \frac{1}{9}\left\{\sin\frac{\pi}{2} - \sin 0\right\}\right]$

$= \left[\frac{1}{3}\left(2+\frac{\pi^2}{12}\right)-\frac{2}{3}\times 0\right]-2\ \left[(0-0)+\frac{1}{9}(1-0)\right]$
$= \frac{2}{3}+\frac{\pi^2}{36}-\frac{2}{9}$
$= \frac{\pi^2}{36}+\frac{4}{9}$
$= \frac{1}{36}\left(\pi^2+16\right)$

29. The given differential equation is,

$(y^2 - 2xy)\ dx = (x^2 - 2xy)\ dy$
$\frac{dy}{dx}=\frac{y^2-2xy}{x^2-2xy}$
This is a homogeneous differential equation
Putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, we have,
$v+x\frac{dv}{dx}=\frac{v^2x^2-2xvx}{x^2-2xvx}$
$v+x\frac{dv}{dx}=\frac{v^2-2v}{1-2v}$
$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}^2-2\mathrm{v}}{1-2\mathrm{v}}-\mathrm{v}$
$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}^2-2\mathrm{v}-\mathrm{v}+2\mathrm{v}^2}{1-2\mathrm{v}}$
$\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{3\mathrm{v}^2-3\mathrm{v}}{1-2\mathrm{v}}$
$\frac{1-2v}{3\left(v^2-v\right)}dv=\frac{dx}{x}$
$\frac{-(2v-1)}{3\left(v^2-v\right)}dv=\frac{dx}{x}$
$\frac{(2v-1)}{\left(v^2-v\right)}dv=-3\frac{dx}{x}$
Integrating both Sides we get,
$\int\frac{(2v-1)}{\left(v^2-v\right)}dv=-3\int\frac{dx}{x}$

$\log |v^2 - v| = -3 \log |x| + \log c$

$v^2 - v = \frac{c}{x^3}$
$\frac{y^2}{x^2}-\frac{y}{x}=\frac{c}{x^3}$
$y^2 - xy = \frac{c}{x}$

$x\ (y^2 - xy) = c$
This is the required differential equation.

OR

The given differential equation is,
$xe^{y/x}-y\sin\left(\frac{y}{x}\right)+x\frac{dy}{dx}\sin\left(\frac{y}{x}\right)=0$
$\Rightarrow\frac{dy}{dx}=\frac{y\sin\left(\frac{y}{x}\right)-xe^{y/x}}{x\sin\left(\frac{y}{x}\right)}$
This is a homogeneous differential equation.
Putting y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, then,we have,
$v+x\frac{dv}{dx}=\frac{v\sin v-e^v}{\sin v}$
$\Rightarrow x\frac{dv}{dx}=\frac{v\sin v-e^v}{\sin v}-v$
$\Rightarrow x\frac{dv}{dx}=-\frac{e^v}{\sin v}$
$\Rightarrow e^{-v}\sin v\ dv = -\frac{dx}{x}$
$\Rightarrow \int e^{-v}\sin v\ dv = -\int\frac{1}{x}\ dx$
$\Rightarrow \frac{e^{-v}}{2}$ (-sin v - cos v) = -log |x| + log C [$\because \int e^{ax}$ sin bx dx = $\frac{e^{ax}}{a^2+b^2}$ (a sin bx - b cos bx)]
$\Rightarrow -\frac{1}{2}e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}$ = -log |x| + log C
$\Rightarrow e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}$ = 2 log |x| - 2 log C
$\Rightarrow e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}$ = log $|x|^2$ - 2 log C ...(ii)
It is given that y(1) = 0 i.e., y = 0 when x = 1. Putting these values in (ii), we get
1 = 0 - 2 log C $\Rightarrow$log C = $-\frac{1}{2}$

Putting log C = $-\frac{1}{2}$ in (ii), we get

$e^{-y/x}\left\{\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right\}$ = log |x|^2 + 1 as the required solution.

30. First, we will convert the given inequations into equations, we obtain the following equations:

2x + y = 18, 3x + 2y = 34

Region represented by 2x + y $\geq$ 18 :

The line 2x + y = 18 meets the coordinate axes at A(9,0) and B(0,18) respectively. By joining these points we obtain the line 2x + y = 18 Clearly (0,0) does not satisfies the inequation 2x + y $\geq$ 18 . So, the region in xy plane which does not contain the origin represents the solution set of the inequation 2x + y $\geq$ 18.

Region represented by 3x + 2y $\leq$ 34 :

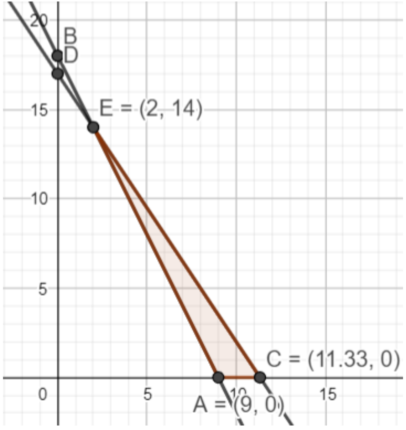
The line 3x + 2y = 34 meets the coordinate axes at

$C\left(\frac{34}{3},0\right)$ and D(0,17) respectively.

By joining these points we obtain the line 3x + 2y = 34 Clearly (0,0) satisfies the inequation 3x + 2y $\leq$ 34 . So, the region containing the origin represents the solution set of the inequation 3x + 2y $\leq$ 34

The corner points of the feasible region are A(9,0)

$C\left(\frac{34}{3},0\right)$ and E(2,14) and feasible region is bounded



The values of Z objective function at these corner points are as follows.

Corner point	Z = 50x + 30y
A(9, 0)	$50 \times 9+3 \times 0=450$
$C\left(\frac{34}{3},0\right)$	$50 \times \frac{34}{3}+30 \times 0=\frac{1700}{3}$
E(2, 14)	$50 \times 2+30 \times 14=520$

Therefore, the maximum value of objective function Z is

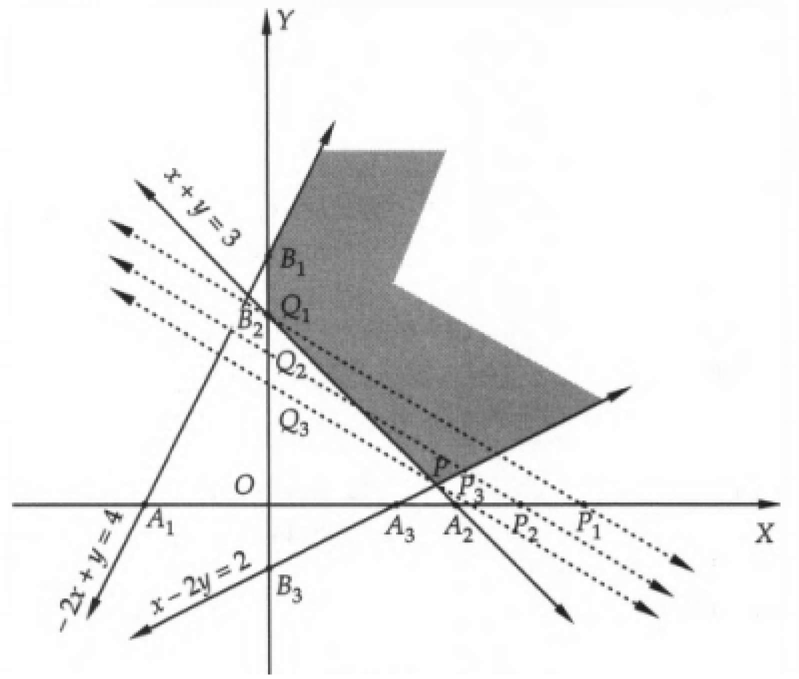
$\frac{1700}{3}$ at the point $\left(\frac{34}{3},0\right)$ Hence, x = $\frac{34}{3}$ and y = 0 is the optimal solution of the given LPP.

Thus, the optimal value of objective function Z is $\frac{1700}{3}$.

OR

Converting the inequations into equations, we obtain the lines -2x + y = 4, x + y = 3 , x - 2y = 2, x = 0 and y = 0.

These lines are drawn on a suitable scale and the feasible region of the LPP is shaded in Fig.



Now, give a value, say 15 equal to (lcm of 3 and 5) to Z to obtain the line 3x + 5y = 15. This line meets the coordinate axes at P_1 (5, 0) and Q_1(0, 3). Join these points by a dotted line. Move this line parallel to itself in the decreasing direction towards the origin so that it passes through only one point of the feasible region. Clearly, P_3Q_3 is such a line passing through the vertex P of the feasible region.

The coordinates of P are obtained by solving the lines x - 2y = 2 and x + y = 3.

Solving these equations, we get x = $\frac{8}{3}$ and y = $\frac{1}{3}$.

Put, x = $\frac{8}{3}$ and y = $\frac{1}{3}$ in Z = 3x + 5y, we get

Z = 3 × $\frac{8}{3}$ = 5 × $\frac{1}{3}$ = $\frac{29}{3}$

Hence, the minimum value of Z is $\frac{29}{3}$ at x = $\frac{8}{3}$, y = $\frac{1}{3}$.

31. Given, x = $a\left(\cos t + \log \tan \frac{t}{2}\right)$ ....(i)

and y = a sin t..........(ii)

Therefore, on differentiating both sides w.r.t t, we get,

$$\frac{dx}{dt} = a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\log \tan \frac{t}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}}\frac{d}{dt}\left(\tan \frac{t}{2}\right)\right]$$ [by using chain rule of derivative]

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2}\frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{\frac{\sin t/2}{\cos t/2}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin \frac{t}{2} \cdot \cos \frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right] \quad [\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= a\left(\frac{1-\sin^2 t}{\sin t}\right)$$

$$\Rightarrow \quad \frac{dx}{dt} = a\left(\frac{\cos^2 t}{\sin t}\right) \left[\because 1 - \sin^2\theta = \cos^2\theta\right]$$ ....(iii)

Again,on differentiating both sides of (ii) w.r.t t, we get,

$\frac{dy}{dt}$ = a cos t...............(iv)

Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)}$ [from Eqs(iii) and (iv)]

$= \frac{a\cos t}{a\cos^2 t} \times \sin t$ = tan t

Therefore,on differentiating both sides of above equation w.r.t x, we get,

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}(\tan t)$

$=\frac{d}{dt}(\tan t)\frac{dt}{dx}\left[\because \frac{d}{dx}f(t)=\frac{d}{dt}f(t)\cdot\frac{dt}{dx}\right]$

$\Rightarrow \quad \frac{d^2y}{dx^2}=\sec^2 t\times\frac{\sin t}{a\cos^2 t}$ [From Eq.(iii)]

$\Rightarrow \quad \frac{d^2y}{dx^2}=\frac{\sin t\sec^4 t}{a}$

Therefore, on putting t = $\frac{\pi}{3}$,we get,

$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{3}}=\frac{\sin\frac{\pi}{3}\times\sec^4\frac{\pi}{3}}{a}=\frac{\frac{\sqrt{3}}{2}\times(2)^4}{a}$

$=\frac{8\sqrt{3}}{a}$

Section D

32. According to the question ,

Given parabola is y^2 = x......(i)

vertex of parabola is (0, 0)

axis of parabola lies along X-axis.

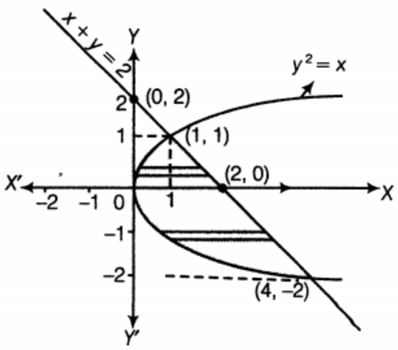
Given equation of line is x + y = 2.......(ii)

For, x + y = 2

x	2	0
y	0	2

So, line passes through the points (2, 0) and (0, 2).

Now, let us sketch the graph of given curve and line as shown below:



On putting x = 2 - y from Eq. (ii) in Eq. (i), we get

$y^2=2-y$

$\Rightarrow y^2+y-2=0$

$\Rightarrow y^2+2y-y-2=0$

$\Rightarrow y(y+2)-1(y+2)=0$

$\Rightarrow (y-1)(y+2)=0$

$\therefore$ y = 1 or -2

When y = 1, then x = 2 - y = 1

When y = -2, then x = 2 - y = 2 - (- 2) = 4

So, points of intersection are (1, 1) and (4, - 2).

Now, required area = $\int_{-2}^{1}\left[x_{(line)}-x_{\text{(parabola)}}\right]dy$

$=\int_{-2}^{1}\left(2-y-y^2\right)dy$

$=\left[2y-\frac{y^2}{2}-\frac{y^3}{3}\right]_{-2}^{1}$

$=\left(2-\frac{1}{2}-\frac{1}{3}\right)-\left(-4-2+\frac{8}{3}\right)$

$=2-\frac{5}{6}+6-\frac{8}{3}$

$=8-\frac{5}{6}-\frac{8}{3}$

$=\frac{48-5-16}{6}$

$=\frac{48-21}{6}$

$= \frac{27}{6}$
$= \frac{9}{2}$ sq units.

33. A = {1, 2, 3, 4, 5} and R = {(a, b) : |a - b| is even}, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

   1. For (a, a), |a - a| = 0 which is even. ∴ R is reflexive.
      If |a - b| is even, then |b - a| is also even. ∴ R is symmetric.
      Now, if |a - b| and |b - c| is even then |a - b + b - c| is even
      ⇒ |a - c| is also even. ∴ R is transitive.
      Therefore, R is an equivalence relation.
   2. Elements of {1, 3, 5} are related to each other.
      Since |1 - 3| = 2, |3 - 5| = 2, |1 - 5| = 4 all are even numbers
      ⇒ Elements of {1, 3, 5} are related to each other.
      Similarly elements of (2, 4) are related to each other.
      Since |2 - 4| = 2 an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).
      Hence no element of {1, 3, 5} is related to any element of {2, 4}.

OR

A = R – {3} and B = R – {1} and $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for f(x_1) = f(x_2)

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-3}{x_2-3}$

$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$= x_1 = x_2$

$\therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y-2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{2y-2-3y+3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

34. We have, A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$\therefore$ BA = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ = 6I

$\therefore$ B^{-1} = $\frac{A}{6} = \frac{1}{6}$ A = $\frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Also, x - y = 3, 2x + 3y + 4z = 17 and y + 2z = 7

$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ [using Eq. (i)]

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore$ x = 2, y = -1 and z = 4

35. $\overrightarrow{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\overrightarrow{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\overrightarrow{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{a}_2 - \overrightarrow{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\overrightarrow{b}_1 \times \overrightarrow{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$(\overrightarrow{a}_2 - \overrightarrow{a}_1)\cdot(\overrightarrow{b}_1 \times \overrightarrow{b}_2) = (3\hat{i} + 3\hat{j} + 3\hat{k})(-9\hat{i} + 3\hat{j} + 9\hat{k}) = -27 + 9 + 27 = 9$

$$d = \left|\frac{(\overrightarrow{a}_2 - \overrightarrow{a}_2)\cdot(\overrightarrow{b}_1 \times \overrightarrow{b}_2)}{\left|\overrightarrow{b}_1 \times \overrightarrow{b}_2\right|}\right|$$

$$= \left|\frac{9}{3\sqrt{19}}\right| = \frac{3}{\sqrt{19}}$$

OR

Let P be the point with position vector $\vec{p} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and M be the image of P in the plane $\overrightarrow{\mathrm{r}} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

In addition, let Q be the foot of the perpendicular from P on to the given plane. So, Q is the midpoint of PM.

Direction ratios of PM are proportional to 2, –1, 1 as PM is normal to the plane and parallel to $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Recall the vector equation of the line passing through the point with position vector$\vec{r}$ and parallel to vector $\overrightarrow{\mathrm{b}}$ is given by

$\overrightarrow{\mathrm{r}} = \overrightarrow{\mathrm{a}} + \lambda\overrightarrow{\mathrm{b}}$

Here,$\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\overrightarrow{\mathrm{b}} = 2\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}$

Hence, the equation of PM is

$\overrightarrow{\mathrm{r}} = (3\hat{\mathrm{i}} + \hat{\mathrm{j}} + 2\hat{\mathrm{k}}) + \lambda(2\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}})$

$\therefore \overrightarrow{\mathrm{r}} = (3 + 2\lambda)\hat{\mathrm{i}} + (1 - \lambda)\hat{\mathrm{j}} + (2 + \lambda)\hat{\mathrm{k}}$

Let the position vector of M be $\overrightarrow{\mathrm{m}}$. As M is a point on this line, for some scalar α, we have

$\Rightarrow \overrightarrow{\mathrm{m}} = (3 + 2\alpha)\hat{\mathrm{i}} + (1 - \alpha)\hat{\mathrm{j}} + (2 + \alpha)\hat{\mathrm{k}}$

Now, let us find the position vector of Q, the midpoint of PM.

Let this be $\overrightarrow{\mathrm{q}}$.

Using the midpoint formula, we have

$\overrightarrow{\mathrm{q}} = \frac{\overrightarrow{\mathrm{p}} + \overrightarrow{\mathrm{m}}}{2}$

$\Rightarrow \overrightarrow{\mathrm{q}} = \frac{[3\hat{\mathrm{i}} + \hat{\mathrm{j}} + 2\hat{\mathrm{k}}] + [(3+2\alpha)\hat{\mathrm{i}} + (1-\alpha)\hat{\mathrm{j}} + (2+\alpha)\hat{\mathrm{k}}]}{2}$

$\Rightarrow \overrightarrow{\mathrm{q}} = \frac{(3+(3+2\alpha))\hat{\mathrm{i}} + (1+(1-\alpha))\hat{\mathrm{j}} + (2+(2+\alpha))\hat{\mathrm{k}}}{2}$

$\Rightarrow \overrightarrow{\mathrm{q}} = \frac{(6+2\alpha)\hat{\mathrm{i}} + (2-\alpha)\hat{\mathrm{j}} + (4+\alpha)\hat{\mathrm{k}}}{2}$

$\therefore \overrightarrow{\mathrm{q}} = (3 + \alpha)\hat{\mathrm{i}} + \frac{(2-\alpha)}{2}\hat{\mathrm{j}} + \frac{(4+\alpha)}{2}\hat{\mathrm{k}}$

This point lies on the given plane, which means this point satisfies the plane equation $\overrightarrow{\mathrm{r}} \cdot (2\hat{\imath} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) = 4$.

$\Rightarrow \left[(3 + \alpha)\hat{\mathrm{i}} + \frac{(2-\alpha)}{2}\hat{\mathrm{j}} + \frac{(4+\alpha)}{2}\hat{\mathrm{k}}\right] \cdot (2\hat{\imath} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) = 4$

$\Rightarrow 2(3 + \alpha) - \left(\frac{2-\alpha}{2}\right)(1) + \left(\frac{4+\alpha}{2}\right)(1) = 4$

$\Rightarrow 6 + 2\alpha + \frac{4+\alpha-(2-\alpha)}{2} = 4$

$\Rightarrow 2\alpha + (1 + \alpha) = -2$

$\Rightarrow 3\alpha = -3$

$\therefore \alpha = -1$

We have the image $\overrightarrow{\mathrm{m}} = (3 + 2\alpha)\hat{\mathrm{i}} + (1 - \alpha)\hat{\mathrm{j}} + (2 + \alpha)\hat{\mathrm{k}}$

$\Rightarrow \overrightarrow{\mathrm{m}}=[3+2(-1)]\hat{\mathrm{i}}+[1-(-1)]\hat{\mathrm{j}}+[2+(-1)]\hat{\mathrm{k}}$

$\therefore \overrightarrow{\mathrm{m}}=\hat{\imath}+2\hat{\jmath}+\hat{\mathrm{k}}$

Therefore, the image is (1, 2, 1)

The foot of the perpendicular $\overrightarrow{\mathrm{q}}=(3+\alpha)\hat{\mathrm{i}}+\frac{(2-\alpha)}{2}\hat{\mathrm{j}}+\frac{(4+\alpha)}{2}\hat{\mathrm{k}}$

$\Rightarrow \overrightarrow{\mathrm{q}}=[3+(-1)]\hat{\mathrm{i}}+\left[\frac{2-(-1)}{2}\right]\hat{\mathrm{j}}+\left[\frac{4+(-1)}{2}\right]\hat{\mathrm{k}}$

$\therefore \overrightarrow{\mathrm{q}}=2\hat{\mathrm{i}}+\frac{3}{2}\hat{\mathrm{j}}+\frac{3}{2}\hat{\mathrm{k}}$

Thus, the position vector of the image is$\hat{\imath}+2\hat{\jmath}+\hat{k}$ and that of the foot of the perpendicular is$2\hat{\imath}+\frac{3}{2}\hat{\jmath}+\frac{3}{2}\hat{k}$

Section E

36. i. Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, E_2 = $\bar{P}\bar{Q}$, E_3 = $\bar{P}Q$, E_4 = $P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

P(E_1) = 0.3 × 0.2 = 0.06, P(E_2) = 0.7 × 0.8 = 0.56, P(E_3) = 0.7 × 0.2 = 0.14, P(E_4) = 0.3 × 0.8 = 0.24

$P\left(\frac{E}{E_1}\right)=0$, $P\left(\frac{E}{E_2}\right)=0$, $P\left(\frac{E}{E_3}\right)=1$, $P\left(\frac{E}{E_4}\right)=1$

P(E) = P(E_1)$\cdot P\left(\frac{E}{E_1}\right)+P\left(E_2\right)\cdot P\left(\frac{E}{E_2}\right)+P\left(E_3\right)\cdot P\left(\frac{E}{E_3}\right)+P\left(E_4\right)\cdot P\left(\frac{E}{E_4}\right)$

= 0.14 + 0.24 + = 0.38

ii. By Bayes' Theorem, $\mathrm{P}\left(\frac{E_3}{E}\right)$ = $\frac{P(E_3)\cdot P\left(\frac{E}{E_3}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$=\frac{0.14}{0.38}=\frac{7}{19}$

NOTE: The four hypotheses form the partition of the sample space and it can be seen that the sum of their probabilities is 1. The hypotheses E_1 and E_2 are actually eliminated as $P\left(\frac{E}{E_1}\right)=P\left(\frac{E}{E_2}\right)=0$

iii. By Bayes' Theorem, $\mathrm{P}\left(\frac{E_4}{E}\right)$ = $\frac{P(E_4)\cdot P\left(\frac{E}{E_4}\right)}{P(E_1)\cdot P\left(\frac{E}{E_1}\right)+P(E_2)\cdot P\left(\frac{E}{E_2}\right)+P(E_3)\cdot P\left(\frac{E}{E_3}\right)+P(E_4)\cdot P\left(\frac{E}{E_4}\right)}$

$=\frac{0.24}{0.38}=\frac{12}{19}$

OR

Let P be the event that the shell fired from A hits the plane and Q be the event that the shell fired from B hits the plane. The following four hypotheses are possible before the trial, with the guns operating independently:

E_1 = PQ, E_2 = $\bar{P}\bar{Q}$, E_3 = $\bar{P}Q$, E_4 = $P\bar{Q}$

Let E = The shell fired from exactly one of them hits the plane.

P(E_1) = 0.3 × 0.2 = 0.06, P(E_2) = 0.7 × 0.8 = 0.56, P(E_3) = 0.7 × 0.2 = 0.14, P(E_4) = 0.3 × 0.8 = 0.24

37. i. Force applied by team A

= $\sqrt{4^2+0^2}$

= 4 N

Force applied by team B

$=\sqrt{(-2)^2+4^2}$

$=\sqrt{4+16}$

$=\sqrt{20}$

$=2\sqrt{5}$ N

Force applied by team C

$=\sqrt{(-3)^2+(-3)^2}$

$=\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

Hence, the force applied by team B is maximum.

So, Team 'B' will win.

ii. Sum of force applied by team A, B and C

= (4 + (-2) + (-3))$\hat{\imath}$ + (0 + 4 + (-3))$\hat{\jmath}$

= $-\hat{\imath}+\hat{\jmath}$

Magnitude of team combine force

$= \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}N$

iii. Force applied by team B

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

$= 2\sqrt{5}$ N

OR

Force applied by team A

$= \sqrt{4^2 + 0^2}$

= 4 N

38. i. $C = 40000h^2 + 5000x^2$

as $x^2h = 250$

$\Rightarrow C = \frac{40000(250)^2}{x^4} + 5000x^2$

ii. $\frac{dC}{dx} = \frac{-160000(250)^2}{x^5} + 10000x$

iii. For minimum cost $\frac{dc}{dx} = 0$

$\Rightarrow 10000x^6 = 250 \times 250 \times 160000$

$\Rightarrow x = 10$

showing $\frac{d^2C}{dx^2} > 0$ at $x = 10$

$\therefore$ cost is minimum when $x = 10$

OR

$\frac{dC}{dx} = \frac{-160000(250)^2}{x^4} + 10000x$

$\frac{dC}{dx} = 0$ gives $x = 10$

$\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in $(0, 10)$.

Hence, cost function is neither increasing nor decreasing for $x > 0$