Series : ZWY7X

SET~3

प्रश्न-पत्र कोड
Q.P. Code **65/7/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे*
*Time allowed : **3** hours*

*अधिकतम अंक : **80***
*Maximum Marks : **80***

	नोट		**NOTE**
(I)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I)	Please check that this question paper contains **23** printed pages.
(II)	प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II)	Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III)	Please check that this question paper contains **38** questions.
(IV)	कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।	(IV)	Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
(V)	इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V)	15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख ग, घ** एवं **ङ**।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।*

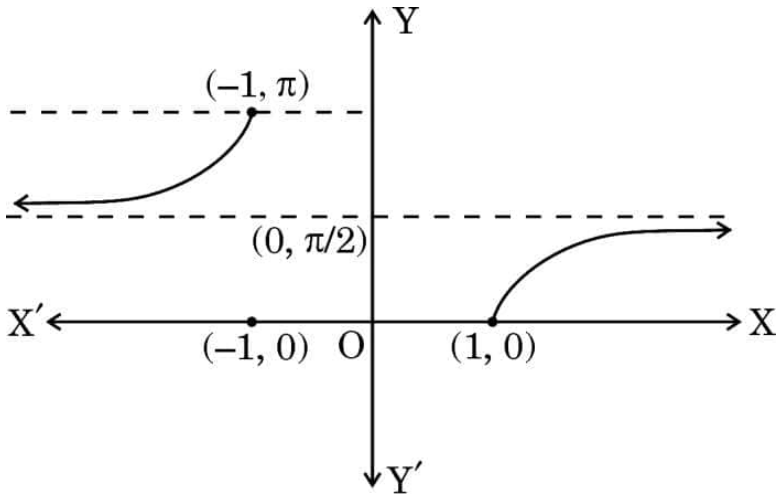
(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. दिया गया ग्राफ दर्शाता है :



(A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$

(C) $y = \tan^{-1} x$ (D) $y = \text{cosec}^{-1} x$

2. माना A, कोटि 3 का एक वर्ग आव्यूह है। यदि $|A| = 5$ है, तो $|\text{adj } A|$ है :

(A) 5 (B) 125

(C) 25 (D) – 5

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

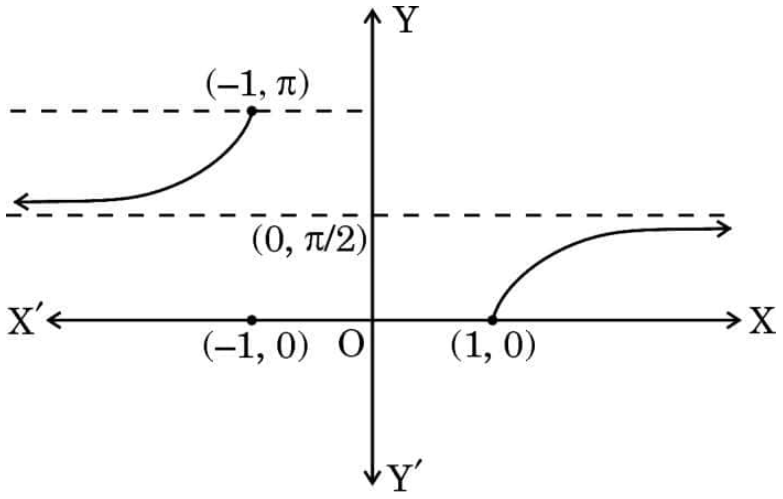
(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The given graph illustrates :



(A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$

(C) $y = \tan^{-1} x$ (D) $y = \text{cosec}^{-1} x$

2. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is :

(A) 5 (B) 125

(C) 25 (D) – 5

3. यदि कोटि 3 के दो वर्ग आव्यूहों A और B के लिए $|A| = 3$ और $|B| = 5$ हैं, तो $|2AB|$ है :

(A) 30 (B) 120

(C) 15 (D) 225

4. 3×3 कोटि के ऐसे संभव आव्यूहों की कुल संख्या कितनी होगी जिनकी प्रत्येक प्रविष्टि $\sqrt{2}$ या $\sqrt{3}$ है ?

(A) 9 (B) 512

(C) 615 (D) 64

5. $f(x) = \cos^{-1} x + \sin x$ का प्रांत है :

(A) R (B) $(-1, 1)$

(C) $[-1, 1]$ (D) ϕ

6. आव्यूह $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ है एक :

(A) अदिश आव्यूह (B) तत्समक आव्यूह

(C) शून्य आव्यूह (D) सममित आव्यूह

7. यदि $f(x) = -2x^8$ है, तो सही कथन है :

(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

8. यदि $f(x) = \begin{cases} 3ax - b & , x > 1 \\ 11 & , x = 1 \\ -5ax - 2b & , x < 1 \end{cases}$

$x = 1$ पर संतत है, तो a और b के मान हैं :

(A) $a = 3,\ b = 5$ (B) $a = 8,\ b = -1$

(C) $a = 1,\ b = -8$ (D) $a = -3,\ b = 5$

3. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30 (B) 120

(C) 15 (D) 225

4. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(A) 9 (B) 512

(C) 615 (D) 64

5. Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(A) R (B) $(-1, 1)$

(C) $[-1, 1]$ (D) ϕ

6. The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an :

(A) scalar matrix (B) identity matrix

(C) null matrix (D) symmetric matrix

7. If $f(x) = -2x^8$, then the correct statement is :

(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

8. If $f(x) = \begin{cases} 3ax - b & , x > 1 \\ 11 & , x = 1 \\ -5ax - 2b & , x < 1 \end{cases}$

is continuous at $x = 1$, then the values of a and b are :

(A) $a = 3,\ b = 5$ (B) $a = 8,\ b = -1$

(C) $a = 1,\ b = -8$ (D) $a = -3,\ b = 5$

9. यदि $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$ है, तो $(x-y)$ का मान है :

(A) 2 या 10 (B) -2 या 10

(C) 2 या -10 (D) -2 या -10

10. एक परिवर्तनशील घन का किनारा 5 cm/s की दर से बढ़ रहा है। घन का पृष्ठीय क्षेत्रफल किस दर से बढ़ रहा है, जबकि किनारा 2 cm लंबा है ?

(A) $24\ cm^2/s$ (B) $120\ cm^2/s$

(C) $12\ cm^2/s$ (D) $5\ cm^2/s$

11. $\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\, dx$ बराबर है :

(A) $x + C$ (B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$

12. यदि $f : R \to R$, $f(x) = 2x - \sin x$ द्वारा परिभाषित फलन है, तो f :

(A) एक ह्रासमान फलन है (B) एक वर्धमान फलन है

(C) का अधिकतम, $x = \frac{\pi}{2}$ पर है (D) का अधिकतम, $x = 0$ पर है

13. एक छात्र दीवार के एक सिरे से दूसरे सिरे तक रस्सियों को एक दूसरे के समांतर बाँधने का प्रयास करता है । यदि एक रस्सी सदिश $3\hat{i} + 15\hat{j} + 6\hat{k}$ के अनुदिश और दूसरी रस्सी सदिश $2\hat{i} + 10\hat{j} + \lambda\hat{k}$ के अनुदिश है, तो λ का मान है :

(A) 6 (B) 1

(C) $\frac{1}{4}$ (D) 4

9. If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x-y)$ is :

(A) 2 or 10 (B) -2 or 10

(C) 2 or -10 (D) -2 or -10

10. Edge of a variable cube increases at the rate of 5 cm/s. The rate at which the surface area of the cube increases when the edge is 2 cm long is :

(A) 24 cm^2/s (B) 120 cm^2/s

(C) 12 cm^2/s (D) 5 cm^2/s

11. $\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}} dx$ is equal to :

(A) $x + C$ (B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$

12. If $f : R \to R$ is defined as $f(x) = 2x - \sin x$, then f is :

(A) a decreasing function (B) an increasing function

(C) maximum at $x = \frac{\pi}{2}$ (D) maximum at $x = 0$

13. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is :

(A) 6 (B) 1

(C) $\frac{1}{4}$ (D) 4

14. $\int \frac{e^{-x}}{16 + 9e^{-2x}} dx$ बराबर है :

(A) $\frac{16}{9} \tan^{-1}(e^{-x}) + C$

(B) $-\frac{1}{12} \tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$

(C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

(D) $-\frac{1}{3} \tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

15. यदि किन्हीं दो सदिशों $\vec{a}$ और $\vec{b}$ के लिए $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ है, तो सदिश $\vec{a}$ और $\vec{b}$:

(A) लंबवत सदिश हैं

(B) एक दूसरे के समान्तर हैं

(C) मात्रक सदिश हैं

(D) संरेख सदिश हैं

16. एक सिक्का उछाला जाता है और 52 ताश की अच्छी तरह से फेंटी हुई गड्डी में से यादृच्छया एक पत्ता निकाला जाता है। सिक्के पर चित और गड्डी से तस्वीर वाला पत्ता आने की प्रायिकता है :

(A) $\frac{2}{13}$

(B) $\frac{3}{26}$

(C) $\frac{19}{26}$

(D) $\frac{3}{13}$

17. यदि A और B दो ऐसी घटनाएँ हैं जिनके लिए $P(B) = \frac{1}{5}$, $P(A \mid B) = \frac{2}{3}$ और $P(A \cup B) = \frac{3}{5}$ है, तो $P(A)$ है :

(A) $\frac{10}{15}$

(B) $\frac{2}{15}$

(C) $\frac{1}{5}$

(D) $\frac{8}{15}$

14. $\int \frac{e^{-x}}{16 + 9e^{-2x}} dx$ is equal to :

(A) $\frac{16}{9} \tan^{-1}(e^{-x}) + C$

(B) $-\frac{1}{12} \tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$

(C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

(D) $-\frac{1}{3} \tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

15. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors

(B) parallel to each other

(C) unit vectors

(D) collinear vectors

16. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

(A) $\frac{2}{13}$

(B) $\frac{3}{26}$

(C) $\frac{19}{26}$

(D) $\frac{3}{13}$

17. If A and B are two events such that $P(B) = \frac{1}{5}$, $P(A \mid B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{5}$, then P(A) is :

(A) $\frac{10}{15}$

(B) $\frac{2}{15}$

(C) $\frac{1}{5}$

(D) $\frac{8}{15}$

18. फलन f(x) के लिए, निम्नलिखित में से कौन-सा सही है ?

(A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(B) $\int_{-a}^{a} f(x)\,dx = 0$, यदि f एक सम फलन है

(C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, यदि f एक विषम फलन है

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a+x)\,dx$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं । दो कथन दिए गए हैं, जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए ।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* $f(x) = \begin{cases} x \sin\frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, $x = 0$ पर संतत है ।

तर्क (R) : जब $x \to 0$, $\sin\frac{1}{x}$, -1 और 1 के बीच एक परिमित मान है ।

20. *अभिकथन (A) :* $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ के मानों का समुच्चय एक रिक्त (शून्य) समुच्चय है ।

तर्क (R) : $\sec^{-1} x$ परिभाषित है, $x \in R - (-1, 1)$ के लिए ।

18. For a function f(x), which of the following holds true ?

(A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function

(C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a+x)\,dx$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at x = 0.

Reason (R) : When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between – 1 and 1.

20. *Assertion (A) :* Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.

खण्ड ख

इस खण्ड में 5 अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. (क) x के सापेक्ष, $\left(\frac{5^x}{x^5}\right)$ का अवकलन कीजिए।

अथवा

(ख) यदि $-2x^2 - 5xy + y^3 = 76$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

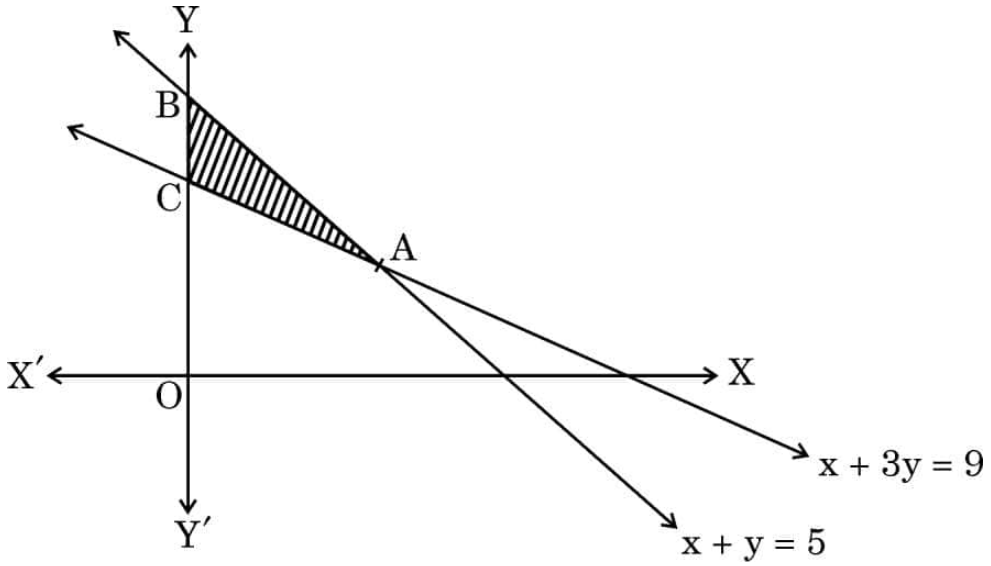
22. यदि $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$ है, तो K का मान ज्ञात कीजिए यदि $A^2 = 6A + KI_2$ है, जहाँ I_2 एक तत्समक आव्यूह है।

23. (क) 10 समान ब्लॉकों को चिह्नित किया गया है। उनमें से दो को '0' से, तीन को '1' से, चार को '2' से और एक को '3' से चिह्नित किया गया है और एक बॉक्स में रखा गया है। यदि X ब्लॉक पर लिखी गई संख्या को दर्शाता है, तो X का प्रायिकता बंटन लिखिए और इसका माध्य परिकलित कीजिए।

अथवा

(ख) 8000 व्यक्तियों के एक गाँव में, 3000 व्यक्ति काम करने के लिए गाँव से बाहर जाते हैं और गाँव में 4000 स्त्रियाँ हैं। इन स्त्रियों में से 30% गाँव के बाहर काम करने के लिए जाती हैं। एक व्यक्ति को यादृच्छया चुना जाता है। चुना गया व्यक्ति, या तो एक स्त्री है या गाँव से बाहर जाकर काम करने वाला एक व्यक्ति है, इसकी प्रायिकता क्या है ?

24. रैखिक प्रोग्रामन समस्या के लिए, उद्देश्य फलन $Z = 5x + 3y$ का न्यूनतमीकरण हेतु छायांकित सुसंगत क्षेत्र नीचे दिए गए आरेख में दिखाया गया है। Z का न्यूनतम मान ज्ञात कीजिए।



(ध्यान दें : आलेख पैमाने अनुसार नहीं है)

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

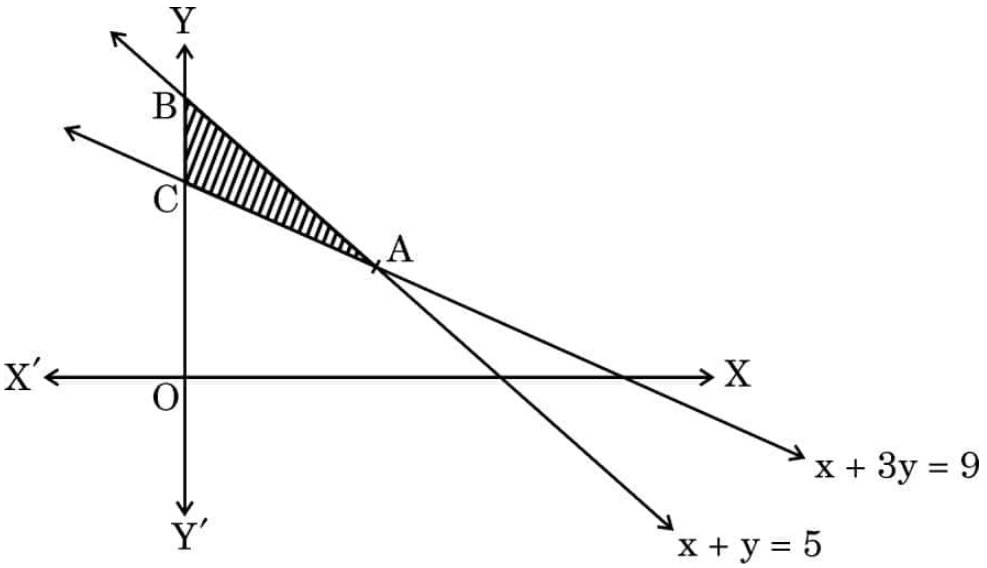
22. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$, then find the value of K if $A^2 = 6A + KI_2$, where I_2 is an identity matrix.

23. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?

24. For a Linear Programming Problem, find min Z = 5x + 3y (where Z is the objective function) for the feasible region shaded in the given figure.



(Note : The figure is not to scale)

25. माना कि $f : A \to B$, $f(x) = \dfrac{x-2}{x-3}$ द्वारा परिभाषित फलन है, जहाँ $A = R - \{3\}$ और $B = R - \{1\}$. फलन के एकैकी-आच्छादी होने पर चर्चा कीजिए।

खण्ड ग

इस खण्ड में 6 लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. रैखिक प्रोग्रामन समस्या में उद्देश्य फलन $Z = 18x + 10y$ का न्यूनतम मान, निम्न व्यवरोधों

$$4x + y \geq 20$$
$$2x + 3y \geq 30$$
$$x, y \geq 0$$

के अंतर्गत ज्ञात कीजिए।

27. (क) सदिश $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ और $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ के योगफल के एक मात्रक सदिश के साथ, सदिश $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ का अदिश गुणनफल 1 है। λ का मान ज्ञात कीजिए।

अथवा

(ख) निम्नलिखित रेखाओं के बीच न्यूनतम दूरी ज्ञात कीजिए :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$
$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$$

28. x के सापेक्ष, $\log(x^x + \text{cosec}^2 x)$ का अवकलन कीजिए।

29. दर्शाइए कि एक निश्चित परिमाप वाले सभी आयतों में, वर्ग का क्षेत्रफल अधिकतम होगा।

30. (क) दर्शाइए कि $f : R \to R$, $f(x) = 4x^3 - 5$, $\forall\ x \in R$ द्वारा परिभाषित फलन एकैकी और आच्छादक है।

अथवा

(ख) माना R, प्राकृत संख्याओं के समुच्चय N, एक संबंध इस प्रकार परिभाषित है

$$R = \{(x, y) : xy \text{ एक प्राकृत संख्या का वर्ग है, } x, y \in N\}.$$

ज्ञात कीजिए कि क्या संबंध R एक तुल्यता संबंध है।

25. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$. Discuss the bijectivity of the function.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints

$$4x + y \geq 20$$
$$2x + 3y \geq 30$$
$$x, y \geq 0$$

find the minimum value of Z.

27. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

OR

(b) Find the shortest distance between the lines :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$$

28. Differentiate $\log(x^x + \text{cosec}^2 x)$ with respect to x.

29. Show that of all the rectangles with a fixed perimeter, the square has the greatest area.

30. (a) Show that the function $f : R \to R$ defined by $f(x) = 4x^3 - 5$, $\forall\ x \in R$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in N\}$. Determine if the relation R is an equivalence relation.

31. (क) मान लीजिए कि $2x + 5y - 1 = 0$ और $3x + 2y - 7 = 0$ दो रेखाओं के समीकरण निरूपित करते हैं जिन पर चींटियाँ जमीन पर चल रही हैं। आव्यूह विधि से, इन चींटियों के रास्तों पर आने वाला एक सामान्य बिन्दु ज्ञात कीजिए।

अथवा

(ख) दिन I पर एक दुकानदार 50 रसायन विज्ञान, 60 भौतिक विज्ञान और 35 गणित की पुस्तकें बेचता है और दिन II पर वह 40 रसायन विज्ञान, 45 भौतिक विज्ञान और 50 गणित की पुस्तकें बेचता है। यदि इन पुस्तकों का विक्रय मूल्य प्रति पुस्तक ₹ 150 (रसायन विज्ञान), ₹ 175 (भौतिक विज्ञान) और ₹ 180 (गणित) हो, तो आव्यूह विधि के प्रयोग से दो दिनों की कुल बिक्री ज्ञात कीजिए। यदि इन सभी पुस्तकों का कुल क्रय मूल्य ₹ 35,000 है, तो दो दिन की बिक्री के बाद कितना लाभ हुआ है ?

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. (क) ज्ञात कीजिए :

$$\int \frac{3x + 1}{(x - 2)^2 (x + 2)} \, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{x}{\cos x + \sin x} \, dx$$

33. (क) रेखा $\frac{2x + 4}{6} = \frac{y + 1}{2} = \frac{-2z + 6}{-4}$ पर एक बिन्दु Q ज्ञात कीजिए जो बिंदु P(1, 2, 3) से $3\sqrt{2}$ की दूरी पर है।

अथवा

(ख) रेखा $\frac{2x - 4}{2} = \frac{y}{2} = \frac{2 - z}{3}$ में बिंदु (–1, 5, 2) का प्रतिबिम्ब ज्ञात कीजिए। दिए गए बिंदु और प्रतिबिम्ब बिंदु को मिलाने वाले रेखाखंड की लम्बाई ज्ञात कीजिए।

31. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{3x + 1}{(x - 2)^2 (x + 2)} \, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\cos x + \sin x} \, dx$$

33. (a) Find the point Q on the line $\frac{2x + 4}{6} = \frac{y + 1}{2} = \frac{-2z + 6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

OR

(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x - 4}{2} = \frac{y}{2} = \frac{2 - z}{3}$. Find the length of the line segment joining the points (given point and the image point).

34. अवकल समीकरण को हल कीजिए :

$(x - \sin y)\, dy + (\tan y)\, dx = 0$, दिया गया है $y(0) = 0$.

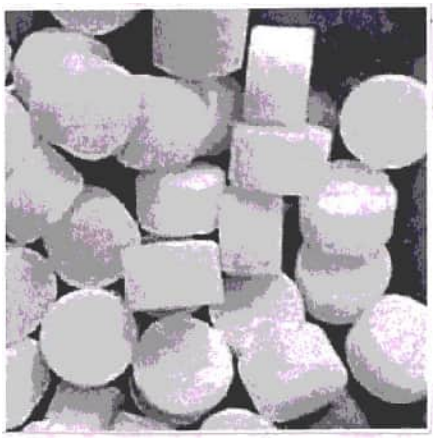
35. एक महिला ने 8 cm त्रिज्या वाली एक वृत्ताकार मेज़ के शीर्ष पर एक सीधी रेखा में एक खरोंच देखी। उसने टेबल टॉप को चार समान चतुर्थांशों में विभाजित किया और पाया कि मूल-बिंदु से गुज़रने वाली खरोंच x-अक्ष की धनात्मक दिशा के साथ वामावर्त दिशा में $\frac{\pi}{4}$ के एक कोण पर झुकी हुई है। समाकलन विधि के प्रयोग से, पहले चतुर्थांश में, x-अक्ष, खरोंच तथा वृत्ताकार मेज़ के शीर्ष से परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. कपूर एक मोमी, रंगहीन तीव्र सुगंध वाला ठोस पदार्थ है जो ऊर्ध्वपातन की प्रक्रिया के माध्यम से खुले में छोड़े जाने पर कमरे के तापमान पर वाष्पित हो जाता है।



(बेलनाकार-आकार की कपूर की गोलियाँ)

एक बेलनाकार कपूर टैबलेट जिसकी ऊँचाई उसकी त्रिज्या (r) के बराबर है, हवा के संपर्क में आने पर वाष्पित हो जाती है, जिससे उसके आयतन में कमी की दर उसके कुल पृष्ठीय क्षेत्रफल के समानुपाती होती है। इस प्रकार अवकल समीकरण $\frac{dV}{dt} = kS$ है, जहाँ V आयतन, S पृष्ठीय क्षेत्रफल है और समय t घंटों में है।

34. Solve the differential equation $(x - \sin y)\,dy + (\tan y)\,dx = 0$, given $y(0) = 0$.

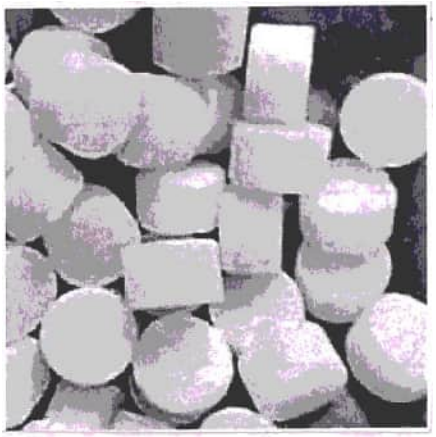
35. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) दिए गए अवकल समीकरण की कोटि व घात ज्ञात कीजिए। *1*

(ii) दिए गए अवकल समीकरण में $V = \pi r^3$ तथा $S = 2\pi r^2$ लेने पर अवकल समीकरण $\frac{dr}{dt} = \frac{2}{3}k$ प्राप्त होता है। इस अवकल समीकरण को हल कीजिए, दिया गया है कि $r(0) = 5$ mm. *1*

(iii) (क) यदि यह दिया गया है कि $t = 1$ घंटे पर $r = 3$ mm है, तो k का मान ज्ञात कीजिए। अत: $r = 0$ mm के लिए t ज्ञात कीजिए। *2*

अथवा

(iii) (ख) यदि यह दिया गया है कि $t = 1$ घंटे पर $r = 1$ mm है, तो k का मान ज्ञात कीजिए। अत: $r = 0$ mm के लिए t ज्ञात कीजिए। *2*

प्रकरण अध्ययन – 2

37. एक अस्पताल में नियमित चिकित्सा जाँच के परिणामों के आधार पर, यह पाया गया कि 1000 लोगों में से 700 बहुत स्वस्थ थे, 200 का स्वास्थ्य औसत बना रहा और 100 का स्वास्थ्य रिकॉर्ड खराब था।

माना A_1 : अच्छे स्वास्थ्य वाले लोग,

A_2 : औसतन स्वास्थ्य वाले लोग,

और A_3 : खराब स्वास्थ्य वाले लोग हैं।

एक महामारी के दौरान, डेटा से पता चलता है कि श्रेणी A_1, A_2 और A_3 के लोगों की बीमारियों से संपर्क करने की संभावना क्रमश: 25%, 35% और 50% है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) एक व्यक्ति की जाँच यादृच्छया की जाती है। इस व्यक्ति के बीमारी के संपर्क में आने की प्रायिकता कितनी है ? *2*

(ii) दिया गया है कि यह व्यक्ति बीमारी के संपर्क में नहीं आया है, तो इस व्यक्ति के श्रेणी A_2 से आने की प्रायिकता क्या है ? *2*

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation. *1*

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. *1*

(iii) (a) If it is given that r = 3 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

OR

(iii) (b) If it is given that r = 1 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

Case Study – 2

37. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

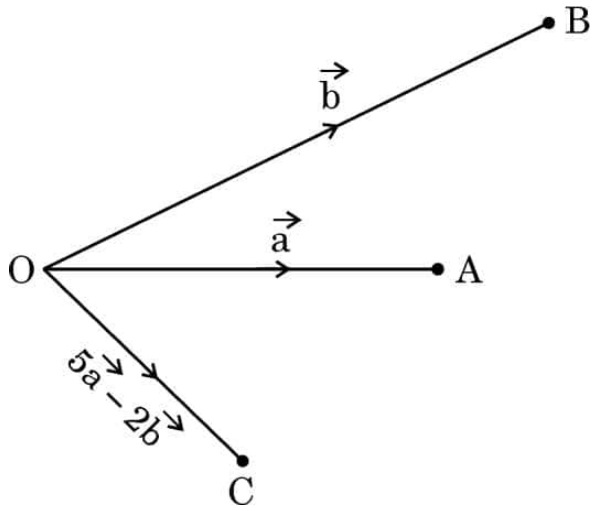
Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease ? *2*

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? *2*

प्रकरण अध्ययन – 3

38. तीन मित्र A, B और C अपने गंतव्य तक पहुँचने के लिए एक ही समय में एक ही स्थान O से तीन अलग-अलग दिशाओं में निकलते हैं। वे सीधे रास्ते पर निकलते हैं और तय करते हैं कि A और B अपने गंतव्य पर पहुँचने के बाद C से उसके पूर्व-निर्धारित गंतव्य पर मिलेंगे, A से C और B से C तक क्रमश: $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ और $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ सीधे रास्ते का अनुसरण करते हुए।



उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) संबंधित सदिशों के साथ उनकी संपूर्ण गति योजना को समझाने के लिए दिए गए चित्र को पूरा कीजिए। *1*

(ii) सदिश $\overrightarrow{AC}$ तथा $\overrightarrow{BC}$ ज्ञात कीजिए। *1*

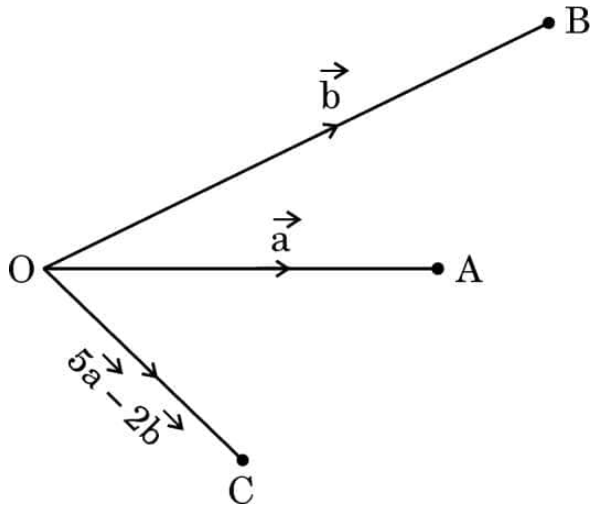
(iii) (क) यदि $\vec{a} \cdot \vec{b} = 1$, O से A की दूरी 1 km और O से B की दूरी 2 km है, तो $\overrightarrow{OA}$ और $\overrightarrow{OB}$ के बीच के कोण को ज्ञात कीजिए। $|\vec{a} \times \vec{b}|$ भी ज्ञात कीजिए। *2*

अथवा

(iii) (ख) यदि $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ तथा $\vec{b} = \hat{j} - \hat{k}$ है, तो एक मात्रक सदिश जो $(\vec{a} + \vec{b})$ और $(\vec{a} - \vec{b})$ दोनों के लम्बवत हो, ज्ञात कीजिए। *2*

Case Study – 3

38. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors. *1*

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. *1*

(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. *2*

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. *2*

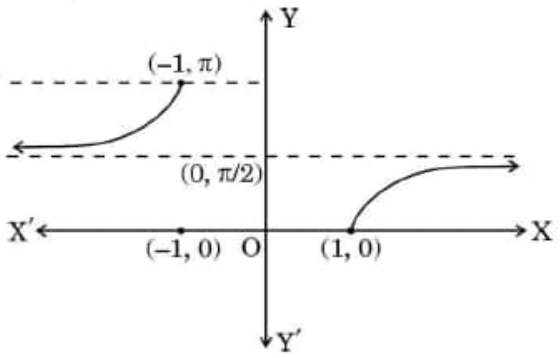
Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary Examination, 2025
SUBJECT: MATHEMATICS (Q.P. CODE – 65/7/3)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks_____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME – 65/7/3

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A ***This section comprises multiple choice questions (MCQs) of 1 mark each.***	
1.	The given graph illustrates :  (A) $y = \sec^{-1} x$ (B) $y = \cot^{-1} x$ (C) $y = \tan^{-1} x$ (D) $y = \operatorname{cosec}^{-1} x$	
Ans	**(A) $\sec^{-1} x$**	**1**
2.	Let A be a square matrix of order 3. If $\lvert A \rvert = 5$, then $\lvert \text{adj } A \rvert$ is : (A) 5 (B) 125 (C) 25 (D) – 5	
Ans	**(C) 25**	**1**
3	If A and B are two square matrices each of order 3 with $\lvert A \rvert = 3$ and $\lvert B \rvert = 5$, then $\lvert 2AB \rvert$ is : (A) 30 (B) 120 (C) 15 (D) 225	
Ans	**(B) 120**	**1**
4.	What is the total number of possible matrices of order 3 × 3 with each entry as $\sqrt{2}$ or $\sqrt{3}$? (A) 9 (B) 512 (C) 615 (D) 64	
Ans	**(B) 512**	**1**
5.	Domain of $f(x) = \cos^{-1} x + \sin x$ is : (A) R (B) (–1, 1) (C) [–1, 1] (D) ϕ	
Ans	**(C) $[-1,1]$**	**1**

6.	The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an : (A) scalar matrix (B) identity matrix (C) null matrix (D) symmetric matrix	
Ans	**(D) symmetric matrix**	**1**
7.	If $f(x) = -2x^8$, then the correct statement is : (A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$ (C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$	
Ans	**(B)** $\mathbf{f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)}$	**1**
8.	If $f(x) = \begin{cases} 3ax - b & , x > 1 \\ 11 & , x = 1 \\ -5ax - 2b & , x < 1 \end{cases}$ is continuous at x = 1, then the values of a and b are : (A) a = 3, b = 5 (B) a = 8, b = –1 (C) a = 1, b = – 8 (D) a = – 3, b = 5	
Ans	**(C) a = 1, b = –8**	**1**
9.	If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of (x – y) is : (A) 2 or 10 (B) – 2 or 10 (C) 2 or – 10 (D) – 2 or – 10	
Ans	**(B) –2 or 10**	**1**

10.	Edge of a variable cube increases at the rate of 5 cm/s. The rate at which the surface area of the cube increases when the edge is 2 cm long is : (A) $24\ cm^2/s$ (B) $120\ cm^2/s$ (C) $12\ cm^2/s$ (D) $5\ cm^2/s$	
Ans	**(B) $120\ cm^2/s$**	**1**
11.	$\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\,dx$ is equal to : (A) $x + C$ (B) $\frac{x^2}{2} + C$ (C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$	
Ans	**(C) $\frac{x^4}{4} + C$**	**1**
12.	If $f : R \to R$ is defined as $f(x) = 2x - \sin x$, then f is : (A) a decreasing function (B) an increasing function (C) maximum at $x = \frac{\pi}{2}$ (D) maximum at $x = 0$	
Ans	**(B) an increasing function**	**1**
13.	A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is : (A) 6 (B) 1 (C) $\frac{1}{4}$ (D) 4	
Ans	**(D) 4**	**1**

14.	$\int \frac{e^{-x}}{16+9e^{-2x}}\,dx$ is equal to : (A) $\frac{16}{9}\tan^{-1}(e^{-x})+C$ (B) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right)+C$ (C) $\tan^{-1}\left(\frac{e^{-x}}{4}\right)+C$ (D) $-\frac{1}{3}\tan^{-1}\left(\frac{e^{-x}}{4}\right)+C$	
Ans	**(B)** $\mathbf{-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right)+C}$	**1**
15.	If $\lvert \vec{a}+\vec{b} \rvert = \lvert \vec{a}-\vec{b} \rvert$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are : (A) orthogonal vectors (B) parallel to each other (C) unit vectors (D) collinear vectors	
Ans	**(A) orthogonal vectors**	**1**
16.	A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is : (A) $\frac{2}{13}$ (B) $\frac{3}{26}$ (C) $\frac{19}{26}$ (D) $\frac{3}{13}$	
Ans	**(B)** $\mathbf{\frac{3}{26}}$	**1**
17.	If A and B are two events such that $P(B)=\frac{1}{5}$, $P(A \mid B)=\frac{2}{3}$ and $P(A \cup B)=\frac{3}{5}$, then P(A) is : (A) $\frac{10}{15}$ (B) $\frac{2}{15}$ (C) $\frac{1}{5}$ (D) $\frac{8}{15}$	

Ans	**(D)** $\frac{8}{15}$	**1**
18.	For a function f(x), which of the following holds true ? (A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ (B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function (C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function (D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a+x)\,dx$	
Ans	**(A)** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

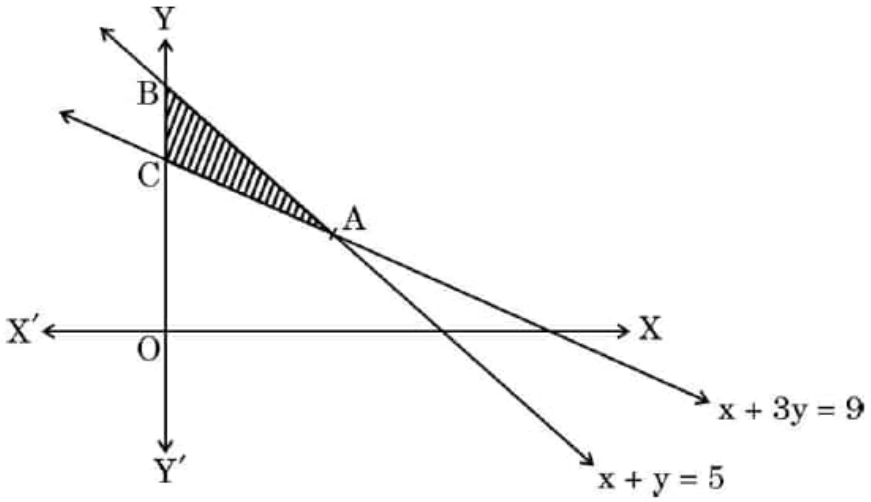
19.	*Assertion (A) :* $f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at $x = 0$. *Reason (R) :* When $x \to 0$, $\sin \frac{1}{x}$ is a finite value between -1 and 1.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
20.	*Assertion (A) :* Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set. *Reason (R) :* $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
	SECTION-B ***This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.***	
21.	(a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x. **OR** (b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.	
Ans	**(a)** Let, $y = \frac{5^x}{x^5} = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = \left(5^x\right)' \cdot x^{-5} + 5^x \cdot \left(x^{-5}\right)'$	**1**

	$= \frac{5^x}{x^5}\log 5 - \frac{5^{x+1}}{x^6}$	1
	Or	
	(b) Differentiating $-2x^2 - 5xy + y^3 = 76$, with respect to 'x'	
	$-4x - 5y - 5x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$	1
	$\Rightarrow \frac{dy}{dx} = \frac{4x+5y}{3y^2-5x}$	1
22.	If $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$, then find the value of K if $A^2 = 6A + KI_2$, where I_2 is an identity matrix.	
Ans	$A^2 = 6A + KI_2 \Rightarrow \begin{bmatrix} 1 & 0 \\ -6 & 25 \end{bmatrix} = \begin{bmatrix} 6+K & 0 \\ -6 & 30+K \end{bmatrix}$	$1\frac{1}{2}$
	$\Rightarrow 6+K=1 \Rightarrow K=-5$, also satisfies $30+K=25$.	$\frac{1}{2}$
23.	(a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.	
	OR	
	(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?	
Ans	**(a) Probability distribution table is:**	

X	0	1	2	3
P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

½

1

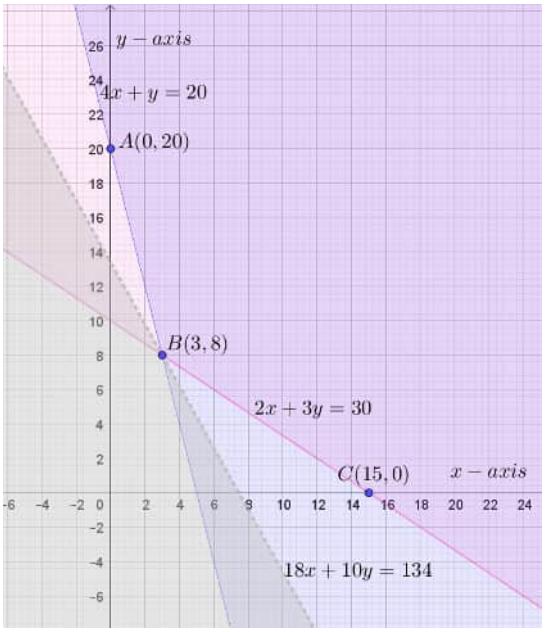
	Mean = E(X) = $\sum p_i x_i = 0\cdot\frac{2}{10}+1\cdot\frac{3}{10}+2\cdot\frac{4}{10}+3\cdot\frac{1}{10}=\frac{14}{10}=\frac{7}{5}$ (or 1.4)	**½**
	OR **(b) A = A randomly chosen person is a woman** **B = A randomly chosen person works outside village.** $P(A)=\frac{4000}{8000}=\frac{1}{2},\ P(B)=\frac{3000}{8000}=\frac{3}{8},\ P(A\cap B)=\frac{1200}{8000}=\frac{3}{20}$	**1½**
	Required probability = $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{3}{8}-\frac{3}{20}=\frac{29}{40}$	**½**
24.	For a Linear Programming Problem, find min Z = 5x + 3y (where Z is the objective function) for the feasible region shaded in the given figure.  (Note : The figure is not to scale)	
Ans		

Corner Points	Value of Z = 5x + 3y
A (3,2)	21
B (0,5)	15
C (0,3)	9

1½

Min (Z) = 9 **½**

25.	Let f : A → B be defined by $f(x)=\frac{x-2}{x-3}$, where A = R – {3} and B = R – {1}. Discuss the bijectivity of the function.	

Ans	**Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3} \Rightarrow x_1 = x_2$, $\therefore$ 'f' is one-one.** **For each $y \in B$, there exists $x = \frac{3y - 2}{y - 1} \in R - \{3\}$, such that $f(x) = y$, $\therefore$ 'f' is onto** **$\Rightarrow$ 'f' is one-one & onto, or 'f' is a bijective function.**	**1** **½** **½**
SECTION-C ***This section comprises 6 Short Answer (SA) type questions of 3 marks each.***		
26.	In the Linear Programming Problem for objective function Z = 18x + 10y subject to constraints $4x + y \geq 20$ $2x + 3y \geq 30$ $x, y \geq 0$ find the minimum value of Z.	
Ans	 **Correct Fig.** **Also, $Z < 134$, does not have any common point with the feasible region,** **$\therefore$ Min$(Z) = 134$ at B (3, 8)**	**1½** **1** **½**

Corner points	Value of Z = 18x + 10y
A (0, 20)	200
B (3, 8)	134
C (15, 0)	270

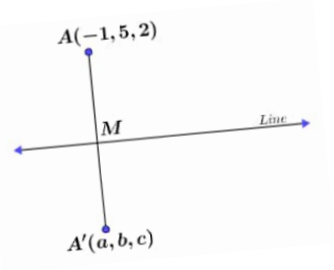
27.	(a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ. **OR** (b) Find the shortest distance between the lines : $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$ $\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$.	
Ans	**(a)** Let $\vec{d} = \vec{b} + \vec{c} = (2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}$ $\hat{d} = \dfrac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$ $\vec{a} \cdot \hat{d} = (\hat{i} - \hat{j} + 2\hat{k}) \cdot \dfrac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$ $\Rightarrow (2+\lambda) + 6 + 4 = \sqrt{(2+\lambda)^2 + 40} \Rightarrow \lambda = -5$ **OR** **(b) The two given lines are parallel with,** $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 4\hat{k}$ **Then** $\vec{a}_2 - \vec{a}_1 = -\hat{i} + \hat{j} + \hat{k}$ **and the parallel vector is** $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ $\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = -5\hat{i} - 4\hat{j} - \hat{k}$ **Shortest Distance** $= \dfrac{\lvert\vec{b} \times (\vec{a}_2 - \vec{a}_1)\rvert}{\lvert\vec{b}\rvert} = \dfrac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$	**1** **½** **1** **½** **½** **1½** **1**

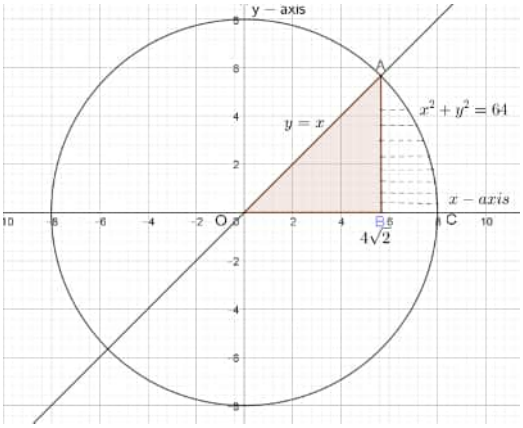
28.	Differentiate $\log(x^x + \text{cosec}^2 x)$ with respect to x.	
Ans	$\frac{d}{dx}\log\left(x^x+\text{cosec}^2 x\right)=\frac{1}{x^x+\text{cosec}^2 x}\frac{d}{dx}\left(e^{x\log x}+\text{cosec}^2 x\right)$ $(\because x^x = e^{x\log x})$ $=\frac{1}{x^x+\text{cosec}^2 x}\left[e^{x\log x}(1+\log x)-2\,\text{cosec}^2 x\cot x\right]$ $=\frac{1}{x^x+\text{cosec}^2 x}\left[x^x(1+\log x)-2\,\text{cosec}^2 x\cot x\right]$	1 $1\frac{1}{2}$ $\frac{1}{2}$
29.	Show that of all the rectangles with a fixed perimeter, the square has the greatest area.	
Ans	**Let P be the perimeter of the rectangle, which is a constant. Also assume 'x' and 'y' be the length and breadth of the rectangle, then** $2(x+y)=P$ **and** $A(\text{Area})=xy=\frac{x}{2}(P-2x)=\frac{1}{2}(Px-2x^2)$ $A'(x)=\frac{1}{2}(P-4x),\ \therefore\ A'(x)=0\Rightarrow x=\frac{P}{4}, y=\frac{P}{4}$ $A''(x)=-2<0$ **at** $x=\frac{P}{4}$, **∴ Area of the rectangle is max. if it is a square.**	$1\frac{1}{2}$ 1 $\frac{1}{2}$
30.	(a) Show that the function $f: R \to R$ defined by $f(x) = 4x^3 - 5$, $\forall\ x \in R$ is one-one and onto. **OR** (b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in N\}$. Determine if the relation R is an equivalence relation.	
Ans	**(a) One-One: Let** $x_1, x_2 \in R$ **such that** $f(x_1)=f(x_2)\Rightarrow 4x_1^3-5=4x_2^3-5\Rightarrow x_1^3=x_2^3\Rightarrow x_1=x_2$, **∴ 'f' is one-one** **Onto:** $x\in R\ (D_f)\Rightarrow x^3\in R\Rightarrow 4x^3-5\in R\Rightarrow f(x)\in R$, **∴** $R_f=\text{Co}-\text{domain}(f)$ **∴ 'f' is an onto function** **⇒ 'f' is one-one & onto both** **OR** **(b) Reflexive: For any** $x\in N$, $x\cdot x=x^2$, **which is square of the natural number 'x'.** $\Rightarrow (x,x)\in R$ **∴ 'R' is a Reflexive relation.**	$1\frac{1}{2}$ $1\frac{1}{2}$ 1

	Symmetric: Let $(x,y) \in R \Rightarrow xy$ is a square of a natural number **$\Rightarrow yx$ is a square of a natural number, $\because xy = yx$.** **$\Rightarrow (y,x) \in R$** **$\therefore$ 'R' is a Symmetric relation.** **Transitive: Let $(x,y),(y,z) \in R \Rightarrow xy = a^2, yz = b^2$ for some $a,b \in N$,** $\therefore \frac{a^2}{y} = x, \frac{b^2}{y} = z \in N$ $\Rightarrow xz = \frac{a^2}{y} \cdot \frac{b^2}{y} = \left(\frac{ab}{y}\right)^2, \frac{ab}{y} \in N$ **$\Rightarrow (x,z) \in R$** **$\therefore$ 'R' is a Transitive relation.** **Hence, R is an Equivalence relation**	**1** **$\frac{1}{2}$** **$\frac{1}{2}$**
31.	(a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants. **OR** (b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?	
Ans	**(a) The system of equations in matrices is:** **$AX = B$, where $A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$** **The solution is given by $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$** **Point common to paths of the ants is $(3,-1)$.** **OR**	**1** **$1\frac{1}{2}$** **$\frac{1}{2}$**

	(a) Let $A=\begin{bmatrix}50 & 60 & 35\\ 40 & 45 & 50\end{bmatrix}$ **Day I / Day II**, $B=\begin{bmatrix}150\\175\\180\end{bmatrix}$ **be the day wise sale and the selling price per subject, matrices respectively.**	**1**
	Total sales day wise $=\begin{bmatrix}50 & 60 & 35\\ 40 & 45 & 50\end{bmatrix}\begin{bmatrix}150\\175\\180\end{bmatrix}=\begin{bmatrix}24,300\\22,875\end{bmatrix}$ **Day I / Day II**	**1**
	Total sales in two days = ₹ 24,300 + ₹ 22,875 = ₹ 47,175	**$\frac{1}{2}$**
	Profit = ₹ 47,175 – ₹ 35,000 = ₹ 12,175.	**$\frac{1}{2}$**
	SECTION-D ***This section comprises 4 Long Answer (LA) type questions of 5 marks each.***	
32.	(a) Find : $\int \frac{3x+1}{(x-2)^2(x+2)}\,dx$ **OR** (b) Evaluate : $\int_0^{\pi/2} \frac{x}{\cos x+\sin x}\,dx$	
Ans	**(a) Using Partial fractions,**	
	$\int \frac{3x+1}{(x-2)^2(x+2)}dx=\frac{5}{16}\int\frac{1}{x-2}dx+\frac{7}{4}\int\frac{1}{(x-2)^2}dx-\frac{5}{16}\int\frac{1}{x+2}dx$	**2$\frac{1}{2}$**
	$=\frac{5}{16}\log\lvert x-2\rvert-\frac{7}{4(x-2)}-\frac{5}{16}\log\lvert x+2\rvert+C$	**2$\frac{1}{2}$**
	or $=\frac{5}{16}\log\left\lvert\frac{x-2}{x+2}\right\rvert-\frac{7}{4(x-2)}+C$	
	Or	

	(b) Let $I=\int_0^{\pi/2}\frac{x}{\cos x+\sin x}dx \Rightarrow I=\int_0^{\pi/2}\frac{\frac{\pi}{2}-x}{\sin x+\cos x}dx \Rightarrow 2I=\frac{\pi}{2}\int_0^{\pi/2}\frac{1}{\cos x+\sin x}dx$	1½
	$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\pi/4\right)}dx=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\pi/4\right)dx$	2
	$\Rightarrow I=\frac{\pi}{4\sqrt{2}}\log\left[\sec\left(x-\pi/4\right)+\tan\left(x-\pi/4\right)\right]\Big]_0^{\frac{\pi}{2}}=\frac{\pi}{4\sqrt{2}}\left[\log\left(\sqrt{2}+1\right)-\log\left(\sqrt{2}-1\right)\right]$	1½
	Or , $I=\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$	
33.	(a) Find the point Q on the line $\frac{2x+4}{6}=\frac{y+1}{2}=\frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3). **OR** (b) Find the image of the point (–1, 5, 2) in the line $\frac{2x-4}{2}=\frac{y}{2}=\frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).	
Ans	**(a) The general point on the line** $(3\lambda-2,2\lambda-1,2\lambda+3)$ **is Q, from some** $\lambda\in R$	2
	$PQ=3\sqrt{2}\Rightarrow (PQ)^2=18\Rightarrow(3\lambda-3)^2+(2\lambda-3)^2+(2\lambda)^2=18$	1
	$17\lambda^2-30\lambda=0\Rightarrow\lambda=0$ **or** $\lambda=\frac{30}{17}$	1
	Thus, the point is $Q(-2,-1,3)$ **or** $Q\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$	1
	Or	
	(b) Let $A'(a,b,c)$ **be the image of the point** $A(-1,5,2)$ **in the given line, also assume 'M' as the point of intersection of** AA' **with the given line, then 'M' is the mid-point of the line segment** AA'	

	 **The Line in the standard form is:** $\frac{x-2}{1}=\frac{y}{2}=\frac{z-2}{-3}$, **then** **M is the point** $(\lambda+2, 2\lambda, -3\lambda+2)$, **for some** $\lambda \in R$ **Direction Ratios of AM are** $\lambda+3, 2\lambda-5, -3\lambda$ $AM \perp Line, \therefore 1(\lambda+3)+2(2\lambda-5)-3(-3\lambda)=0 \Rightarrow \lambda=\frac{1}{2}$ $M\left(\frac{5}{2},1,\frac{1}{2}\right)=M\left(\frac{a-1}{2},\frac{b+5}{2},\frac{c+2}{2}\right) \Rightarrow a=6, b=-3, c=-1$ $\therefore$ **The Image of A in the line is** $A'(6,-3,-1)$ **And,** $AA'=\sqrt{49+64+9}=\sqrt{122}$	1 ½ ½ 1 1 ½ ½
s 34.	Solve the differential equation $(x-\sin y)\,dy + (\tan y)\,dx = 0$, given $y(0) = 0$.	
Ans	**The differential equation can be written as:** $\frac{dx}{dy}+\cot y \cdot x=\cos y$, **which is a linear order differential equation** **Here,** $P=\cot y$, $Q=\cos y$, **I.F. (Integrating Factor)** $= e^{\int \cot y\, dy} = e^{\log \sin y} = \sin y$ **The solution is ,** $x(\sin y)=\int \cos y \cdot \sin y\, dy$ $\Rightarrow x(\sin y)=\frac{(\sin y)^2}{2}+C$, **For** $x=0, y=0$, $C=0$. $\therefore$ **The Particular solution is:** $x\sin y=\frac{\sin^2 y}{2}$ **or** $\sin y=2x$ **or** $y=\sin^{-1}2x$	1 1½ 1 1 ½

35.	A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.	
Ans		

Correct graph — 1

Equation of the circular tabletop: $x^2 + y^2 = 64$ — ½

Equation of line (scratch): $x = y$ — ½

The line and circle intersect at $x = 4\sqrt{2}$ — ½

Area of the shaded region

$$= \int_0^{4\sqrt{2}} x\,dx + \int_{4\sqrt{2}}^{8} \sqrt{64 - x^2}\,dx$$ — 1

$$= \frac{x^2}{2}\Big]_0^{4\sqrt{2}} + \left[\frac{x}{2}\sqrt{64 - x^2} + 32\sin^{-1}\frac{x}{8}\right]_{4\sqrt{2}}^{8}$$ — 1

$$= \frac{32}{2} + 32\sin^{-1}1 - 2\sqrt{2}\cdot 4\sqrt{2} - 32\sin^{-1}\frac{1}{\sqrt{2}}$$

$$= 16 + 16\pi - 16 - 8\pi = 8\pi \text{ cm}^2$$ — ½

36.

Case Study – 1

Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation. *1*

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm. *1*

(iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm. *2*

OR

(iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm. *2*

Ans

(i) Order = 1, Degree = 1 ½ + ½

(ii) Separating the variable and integrating, $\int dr = \frac{2k}{3}\int dt \Rightarrow r = \frac{2}{3}kt + C$ ½

Putting $t = 0, r = 5$**, we get** $C = 5$

$$r = \frac{2}{3}kt + 5$$ ½

(iii) (a) Putting $r = 3, t = 1$**,** $3 = \frac{2}{3}k(1) + 5 \Rightarrow k = -3$ 1

$r = -2t + 5$**, For** $r = 0$**,** $t = \frac{5}{2}$ **hrs or 2.5 hours** 1

Or

(iii) (b) Putting $r = 1, t = 1$**,** $1 = \frac{2}{3}k + 5 \Rightarrow k = -6$ 1

$\therefore\ r = -4t + 5$**, For** $r = 0$**,** $t = \frac{5}{4}$ **hrs or 1.25 hours** 1

Case Study – 2

37. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease ? *2*

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? *2*

Ans

(i) Let A: Person contracted the disease

$$P(A) = P(A_1)\cdot P(A \mid A_1) + P(A_2)\cdot P(A \mid A_2) + P(A_3)\cdot P(A \mid A_3)$$

$$= \frac{7}{10}\left(\frac{25}{100}\right) + \frac{2}{10}\left(\frac{35}{100}\right) + \frac{1}{10}\left(\frac{50}{100}\right) \quad 1\frac{1}{2}$$

$$= \frac{295}{1000} = 0.295 \text{ or } \left(\frac{59}{200}\right) \quad \frac{1}{2}$$

(ii) $$P(A_2 \mid \bar{A}) = \frac{P(A_2)\cdot P(\bar{A}/A_2)}{P(A_1)\cdot P(\bar{A}/A_1) + P(A_2)\cdot P(\bar{A}/A_2)}$$

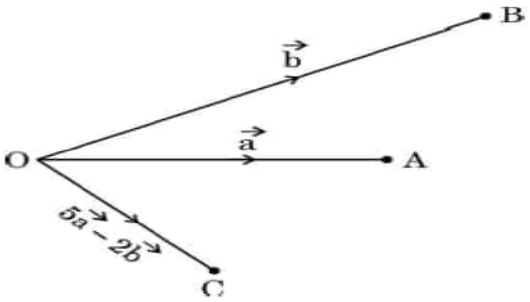
$$= \frac{\frac{2}{10}\times\frac{65}{100}}{\frac{7}{10}\times\frac{75}{100} + \frac{2}{10}\times\frac{65}{100} + \frac{1}{10}\times\frac{50}{100}} \quad 1\frac{1}{2}$$

$$= \frac{2\times 13}{7\times 15 + 2\times 13 + 1\times 10} = \frac{26}{141} \quad \frac{1}{2}$$

38.

Case Study – 3

Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors. *1*

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. *1*

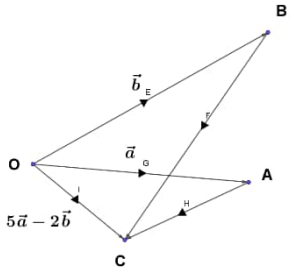
(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. *2*

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. *2*

Ans

(i) The Complete figure of their entire movement plan is:



1

(ii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\vec{a} - 2\vec{b}$, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 5\vec{a} - 3\vec{b}$ 1

(iii) (a) we are given: $|\vec{a}| = 1, |\vec{b}| = 2$, **assuming ' θ ' as the angle between** $\overrightarrow{OA}$ **and** $\overrightarrow{OB}$.

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\frac{1}{1 \times 2} = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$ 1

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 1(2)\frac{\sqrt{3}}{2} = \sqrt{3}$$ 1

	Or	
	(iii) (b) $\vec{a}+\vec{b}=2\hat{i}+3\hat{k}$, $\vec{a}-\vec{b}=2\hat{i}-2\hat{j}+5\hat{k}$, let $\vec{c}$ be $\perp$ to both $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$	½
	Then, $\vec{c}=(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 3\\ 2 & -2 & 5\end{vmatrix}=6\hat{i}-4\hat{j}-4k$ **and** $\lvert\vec{c}\rvert=\sqrt{68}$	1
	The required unit vector is, $\hat{c}=\frac{1}{2\sqrt{17}}(6\hat{i}-4\hat{j}-4\hat{k})=\frac{1}{\sqrt{17}}(3\hat{i}-2\hat{j}-2\hat{k})$	½