Series : ZXW5Y

SET~3

प्रश्न-पत्र कोड
Q.P. Code **65/5/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे*
*Time allowed : **3** hours*

*अधिकतम अंक : **80***
*Maximum Marks : **80***

	नोट		**NOTE**
(I)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I)	Please check that this question paper contains **23** printed pages.
(II)	प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II)	Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III)	Please check that this question paper contains **38** questions.
(IV)	**कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**	(IV)	**Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
(V)	इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V)	15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ का मुख्य मान है :

(A) $-\frac{\pi}{3}$ (B) $-\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

2. यदि $A = [a_{ij}]$ कोटि 3×3 का एक विकर्ण आव्यूह है जिसमें $a_{11} = 1$, $a_{22} = 5$ तथा $a_{33} = -2$ है, तो $|A|$ है :

(A) 0 (B) –10

(C) 10 (D) 1

3. यदि $A = kB$ है, जहाँ A और B कोटि n के दो वर्ग आव्यूह हैं और k एक अदिश है, तो :

(A) $|A| = k|B|$ (B) $|A| = k^n|B|$

(C) $|A| = k + |B|$ (D) $|A| = |B|^k$

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is :

(A) $-\frac{\pi}{3}$ (B) $-\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

2. If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(A) 0 (B) –10

(C) 10 (D) 1

3. If A = kB, where A and B are two square matrices of order n and k is a scalar, then :

(A) $|A| = k|B|$ (B) $|A| = k^n|B|$

(C) $|A| = k + |B|$ (D) $|A| = |B|^k$

4. यदि $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

$x = 0$ पर संतत है, तो a का मान है :

(A) 1 (B) -1

(C) ± 1 (D) 0

5. यदि $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$ है, तो x का मान है :

(A) 3 (B) 7

(C) ± 7 (D) ± 3

6. यदि $P(A \cup B) = 0{\cdot}9$ तथा $P(A \cap B) = 0{\cdot}4$ है, तो $P(\overline{A}) + P(\overline{B})$ है :

(A) 0·3 (B) 1

(C) 1·3 (D) 0·7

7. यदि $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ है, तो A^3 है :

(A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

8. माना A और B उपयुक्त कोटि के दो आव्यूह हैं। तब, निम्नलिखित में से कौन-सा सही ***नहीं*** है ?

(A) $(A')' = A$ (B) $(kA)' = kA'$, k एक अदिश है

(C) $(A' + B')' = A + B$ (D) $(AB)' = A'B'$

9. वक्र $y = x|x|$, x-अक्ष, $x = -2$ और $x = 2$ से घिरे क्षेत्र का क्षेत्रफल है :

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$

(C) 0 (D) 8

4. If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is :

(A) 1 (B) -1

(C) ± 1 (D) 0

5. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(A) 3 (B) 7

(C) ± 7 (D) ± 3

6. If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is :

(A) 0·3 (B) 1

(C) 1·3 (D) 0·7

7. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

(A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

8. Let A and B be two matrices of suitable orders. Then, which of the following is ***not*** correct ?

(A) $(A')' = A$ (B) $(kA)' = kA'$, k is a scalar

(C) $(A' + B')' = A + B$ (D) $(AB)' = A'B'$

9. The area of the region enclosed between the curve $y = x|x|$, x-axis, $x = -2$ and $x = 2$ is :

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$

(C) 0 (D) 8

10. यदि $f(x) = \{[x], x \in R\}$ एक महत्तम पूर्णांक फलन है, तो निम्नलिखित में से सही कथन है :

(A) $x = 2$ पर f संतत है परंतु अवकलनीय नहीं है।

(B) $x = 2$ पर f न तो संतत है और न ही अवकलनीय है।

(C) $x = 2$ पर f संतत और अवकलनीय है।

(D) $x = 2$ पर f संतत नहीं है परंतु अवकलनीय है।

11. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} \, dx$ बराबर है :

(A) $2(\sin x + x \cos \theta) + C$ (B) $2(\sin x - x \cos \theta) + C$

(C) $2(\sin x + \sin \theta) + C$ (D) $2(\sin x - x \sin \theta) + C$

12. $\int_0^1 \frac{2x}{5x^2 + 1} \, dx$ बराबर है :

(A) $\frac{1}{5} \log 6$ (B) $\frac{1}{5} \log 5$

(C) $\frac{1}{2} \log 6$ (D) $\frac{1}{2} \log 5$

13. वक्र $y = -x^3 + 3x^2 + 8x - 20$ की प्रवणता जिस बिंदु पर अधिकतम है, वह है :

(A) $(1, -10)$ (B) $(1, 10)$

(C) $(10, 1)$ (D) $(-10, 1)$

14. अवकल समीकरण

$$\frac{dx}{dy} = \frac{-(1 + \sin x)}{x + y \cos x}$$

का समाकलन गुणक है :

(A) $\log \cos x$ (B) $1 + \sin x$

(C) $e^{(1 + \sin x)}$ (D) $e^{\log \cos x}$

10. If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is :

(A) f is continuous but not differentiable at x = 2.

(B) f is neither continuous nor differentiable at x = 2.

(C) f is continuous as well as differentiable at x = 2.

(D) f is not continuous but differentiable at x = 2.

11. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta} \, dx$ is equal to :

(A) $2(\sin x + x \cos \theta) + C$ (B) $2(\sin x - x \cos \theta) + C$

(C) $2(\sin x + \sin \theta) + C$ (D) $2(\sin x - x \sin \theta) + C$

12. $\int_0^1 \frac{2x}{5x^2 + 1} \, dx$ is equal to :

(A) $\frac{1}{5} \log 6$ (B) $\frac{1}{5} \log 5$

(C) $\frac{1}{2} \log 6$ (D) $\frac{1}{2} \log 5$

13. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

(A) (1, –10) (B) (1, 10)

(C) (10, 1) (D) (–10, 1)

14. The integrating factor of the differential equation

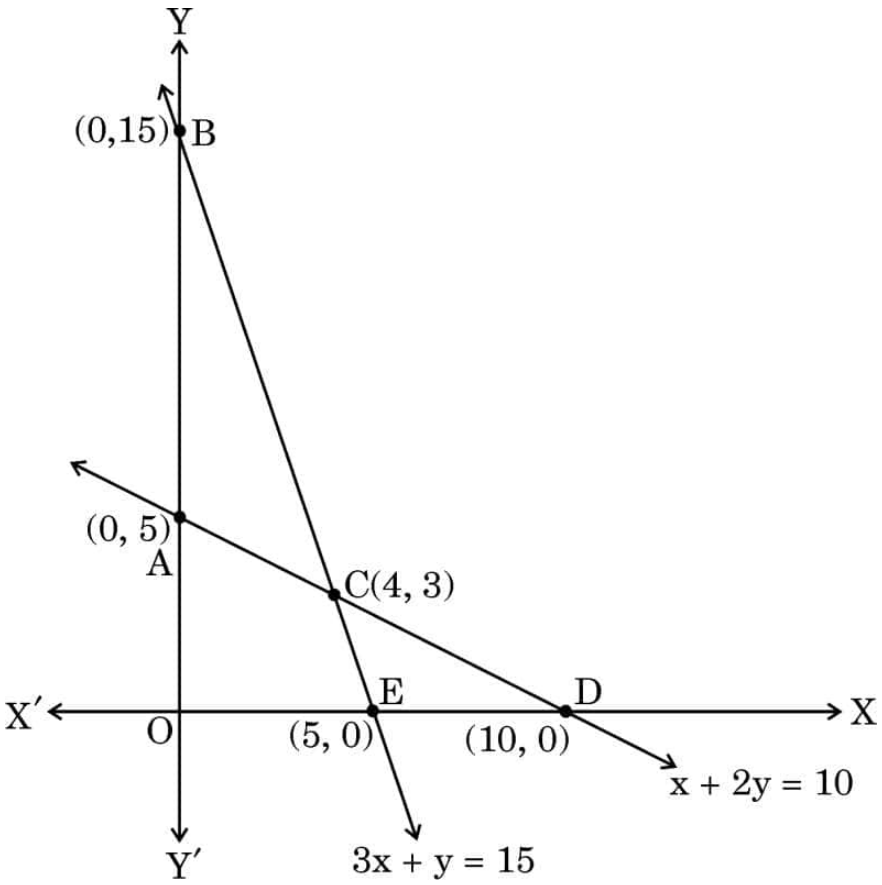
$\frac{dx}{dy} = \frac{-(1 + \sin x)}{x + y \cos x}$ is :

(A) $\log \cos x$ (B) $1 + \sin x$

(C) $e^{(1 + \sin x)}$ (D) $e^{\log \cos x}$

15. एक रैखिक प्रोग्रामन समस्या (LPP) के लिए, दिया गया उद्देश्य फलन $Z = 3x + 2y$ निम्न व्यवरोधों के अंतर्गत है :

$$x + 2y \leq 10$$
$$3x + y \leq 15$$
$$x, y \geq 0$$



सही सुसंगत क्षेत्र है :

(A) ABC (B) AOEC

(C) CED (D) खुला अपरिबद्ध क्षेत्र BCD

16. अवकल समीकरण

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$$

की कोटि और घात का योगफल है :

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

17. यदि $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ है और $|\vec{a}| = 3|\vec{b}|$ है, तो $|\vec{a}|$ और $|\vec{b}|$ के मान क्रमश: हैं :

(A) 48 और 16 (B) 3 और 1

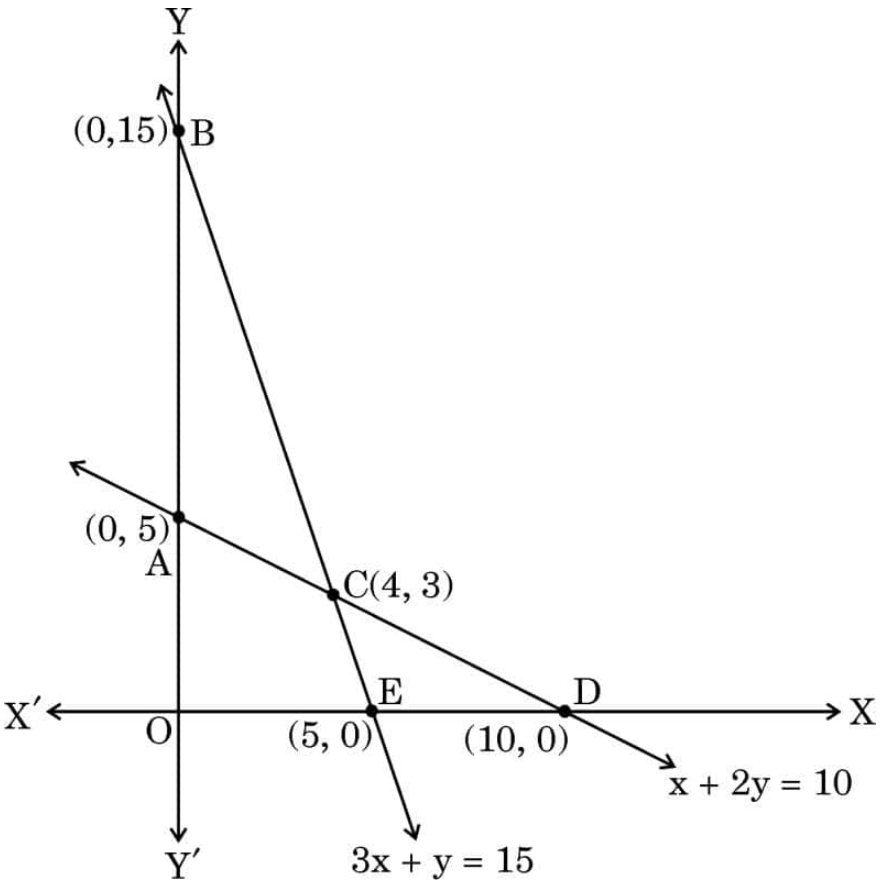
(C) 24 और 8 (D) 6 और 2

15. For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints :

$x + 2y \leq 10$

$3x + y \leq 15$

$x, y \geq 0$



The correct feasible region is :

(A) ABC (B) AOEC

(C) CED (D) Open unbounded region BCD

16. The sum of the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$$ is :

(A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4

17. The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are :

(A) 48 and 16 (B) 3 and 1

(C) 24 and 8 (D) 6 and 2

18. मान लीजिए $\vec{a}$ एक स्थिति सदिश है जिसका शीर्ष बिंदु $(2, -3)$ है। यदि $\overrightarrow{AB} = \vec{a}$ है, जहाँ बिंदु A के निर्देशांक $(-4, 5)$ हैं, तो बिंदु B के निर्देशांक हैं :

(A) $(-2, -2)$ (B) $(2, -2)$ (C) $(-2, 2)$ (D) $(2, 2)$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं, जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।*

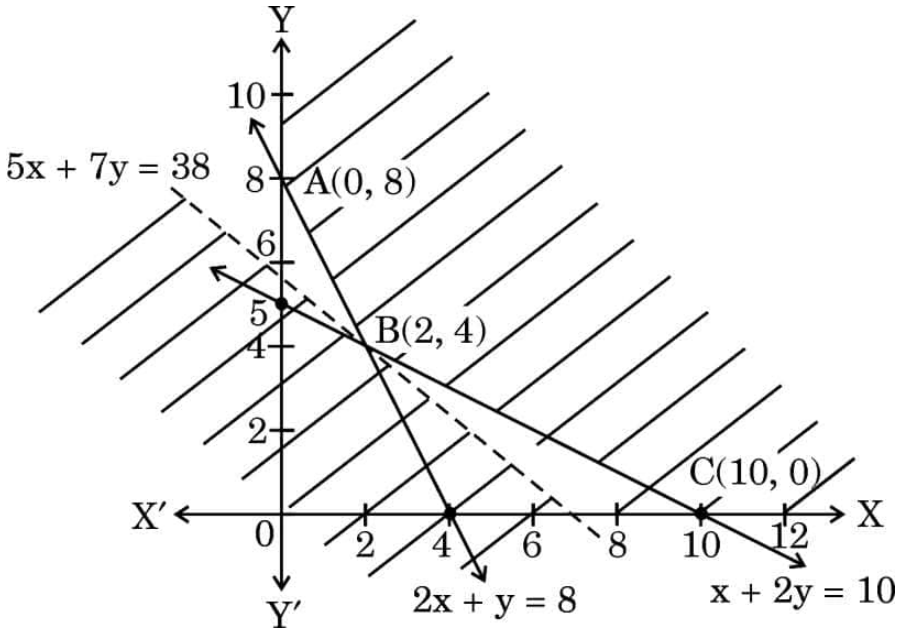
(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

19. *अभिकथन (A) :* आलेख का छायांकित भाग दिए गए रैखिक प्रोग्रामन समस्या (LPP) का सुसंगत क्षेत्र निरूपित करता है।



$Z = 50x + 70y$ का न्यूनतमीकरण

निम्न व्यवरोधों के अंतर्गत :

$2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$

यहाँ Z का न्यूनतम मान B(2, 4) पर 380 है।

तर्क (R) : $50x + 70y < 380$ द्वारा निरूपित क्षेत्र का कोई बिंदु सुसंगत क्षेत्र से साझा नहीं है।

18. Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are :

(A) (– 2, – 2) (B) (2, – 2) (C) (– 2, 2) (D) (2, 2)

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

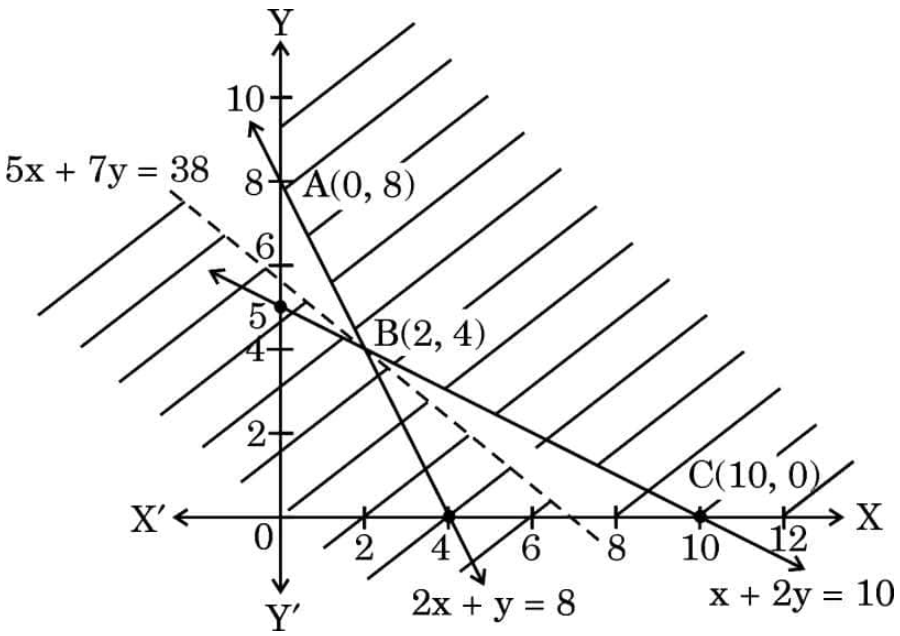
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min Z = 50x + 70y

subject to constraints

$2x + y \geq 8$, $x + 2y \geq 10$, $x, y \geq 0$

Z = 50x + 70y has a minimum value = 380 at B(2, 4).

Reason (R) : The region representing 50x + 70y < 380 does not have any point common with the feasible region.

20. *अभिकथन (A) :* माना $A = \{x \in R : -1 \leq x \leq 1\}$. यदि $f : A \to A$, $f(x) = x^2$ द्वारा परिभाषित है, तो f आच्छादक फलन नहीं है।

तर्क (R) : यदि $y = -1 \in A$ है, तो $x = \pm \sqrt{-1} \notin A$.

खण्ड ख

इस खण्ड में 5 अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. $\sin^{-1}(x^2 - 3)$ का प्रांत ज्ञात कीजिए।

22. मान लीजिए किसी धातु के खोखले गोले का आयतन स्थिर है। यदि आंतरिक त्रिज्या 2 cm/s की दर से बढ़ती है, तो बाहरी त्रिज्या की वृद्धि की दर ज्ञात कीजिए जब त्रिज्याएँ क्रमश: 2 cm और 4 cm हैं।

23. एक व्यक्ति एक सीधी तार, जिसके सिरों के निर्देशांक A (4, 1, – 2) तथा B (6, 2, – 3) हैं, पर दो लालटेन इस प्रकार लटकाना चाहता है कि ये लालटेन वाले बिंदु तार AB को सम-त्रिभाजित करें। उन बिंदुओं के निर्देशांक ज्ञात कीजिए जिन पर इन दोनों लालटेनों को लटकाना है।

24. (क) x के सापेक्ष, $\frac{\sin x}{\sqrt{\cos x}}$ का अवकलन कीजिए।

अथवा

(ख) यदि $y = 5 \cos x - 3 \sin x$ है, तो सिद्ध कीजिए कि $\frac{d^2y}{dx^2} + y = 0$ है।

25. (क) एक सदिश ज्ञात कीजिए जिसका परिमाण 5 है, तथा जो दोनों सदिशों $3\hat{i} - 2\hat{j} + \hat{k}$ और $4\hat{i} + 3\hat{j} - 2\hat{k}$ के लंबवत है।

अथवा

(ख) माना $\vec{a}, \vec{b}$ और $\vec{c}$ तीन ऐसे सदिश हैं, जिनके लिए $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ और $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$ है। दर्शाइए कि $\vec{b} = \vec{c}$.

20. *Assertion (A) :* Let $A = \{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm \sqrt{-1} \notin A$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $\sin^{-1}(x^2 - 3)$.

22. Let the volume of a metallic hollow sphere be constant. If the inner radius increases at the rate of 2 cm/s, find the rate of increase of the outer radius when the radii are 2 cm and 4 cm respectively.

23. A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

24. (a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x.

OR

(b) If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

25. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$.

OR

(b) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.

खण्ड ग

इस खण्ड में 6 लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. वे अंतराल ज्ञात कीजिए जिनमें फलन $f(x) = \sin 3x - \cos 3x,\ 0 < x < \frac{\pi}{2}$ निरंतर वर्धमान है।

27. (क) यदि $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ है, जिसमें $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$ है, तो $\vec{a}$ और $\vec{b}$ के बीच का कोण ज्ञात कीजिए।

अथवा

(ख) यदि मात्रक सदिश $\vec{a}$ और $\vec{b}$ के बीच का कोण θ है, तो सिद्ध कीजिए कि $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

28. अवकल समीकरण

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$$

का हल ज्ञात कीजिए।

29. (क) एक विद्यार्थी द्वारा एक रंग भरने वाली पुस्तक के खरीदने की प्रायिकता 0·7 और रंगों के बॉक्स को खरीदने की प्रायिकता 0·2 है। उसके एक रंग भरने वाली पुस्तक को खरीदने की प्रायिकता, यह जानते हुए कि उसने रंगों का बॉक्स खरीद लिया है, 0·3 है। प्रायिकता ज्ञात कीजिए कि वह विद्यार्थी :

(i) दोनों, रंग भरने वाली पुस्तक और रंगों का बॉक्स, खरीदती है।

(ii) रंगों का बॉक्स खरीदती है, यह जानते हुए कि उसने रंग भरने वाली पुस्तक खरीद ली है।

अथवा

(ख) एक व्यक्ति के पास फलों का एक बॉक्स है जिसमें 6 सेब और 4 संतरे हैं। वह इस बॉक्स से यादृच्छया एक फल, एक-एक करके तीन बार निकालता है, हर बार निकाले गए फल को पुन: बॉक्स में रख दिया जाता है। ज्ञात कीजिए :

(i) निकाले गए संतरों की संख्या का प्रायिकता बंटन।

(ii) यादृच्छिक चर (संतरों की संख्या) की प्रत्याशा।

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Find the interval/intervals in which the function $f(x) = \sin 3x - \cos 3x$, $0 < x < \frac{\pi}{2}$ is strictly increasing.

27. (a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

28. Solve the differential equation

$$x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x.$$

29. (a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student :

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find :

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

30. (क) ज्ञात कीजिए :

$$\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_1^4 \left(|x-2|+|x-4|\right)\,dx$$

31. निम्नलिखित रैखिक प्रोग्रामन समस्या (LPP) के लिए अधिकतम वाला/वाले बिन्दु ज्ञात कीजिए।

$Z = 5x + 10y$

निम्न व्यवरोधों के अंतर्गत

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x, y \geq 0$

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. $y = 1 + |x + 1|$, $x = -2$, $x = 2$ और $y = 0$ के बीच के क्षेत्र का रफ ग्राफ बनाइए। समाकलन से, इस क्षेत्र का क्षेत्रफल भी ज्ञात कीजिए।

33. तीन छात्र एक रेसिंग ट्रैक पर इस प्रकार दौड़ते हैं कि उनकी गतियों का योगफल 6 km/h हो जाता है। हालाँकि, तीसरे धावक की गति के दुगुने को पहले धावक की गति में जोड़ने पर परिणाम गति 7 km/h हो जाती है। पहले धावक की गति के तीन गुने को दूसरे और तीसरे धावकों की मूल गतियों में जोड़ने से परिणाम गति 12 km/h हो जाती है। आव्यूह विधि के प्रयोग से, प्रत्येक धावक की मूल गति ज्ञात कीजिए।

30. (a) Find :

$$\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$$

OR

(b) Evaluate :

$$\int_1^4 \left(|x-2|+|x-4|\right)\,dx$$

31. In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$

subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x, y \geq 0$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. In a rough sketch, mark the region bounded by $y = 1 + |x + 1|$, $x = -2$, $x = 2$ and $y = 0$. Using integration, find the area of the marked region.

33. Three students run on a racing track such that their speeds add up to 6 km/h. However, double the speed of the third runner added to the speed of the first results in 7 km/h. If thrice the speed of the first runner is added to the original speeds of the other two, the result is 12 km/h. Using matrix method, find the original speed of each runner.

34. (क) एक धनात्मक स्थिरांक 'a' के लिए, $\left(t+\frac{1}{t}\right)^a$ के सापेक्ष $a^{t+\frac{1}{t}}$ का अवकलन कीजिए, जहाँ t एक शून्येतर वास्तविक संख्या है।

अथवा

(ख) यदि $y^x + x^y + x^x = a^b$ है, जहाँ a और b स्थिरांक हैं, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

35. (क) रेखा $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$ पर बिंदु (1, 1, 4) से डाले गए लंब का पाद ज्ञात कीजिए।

अथवा

(ख) रेखा $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ का वह बिंदु ज्ञात कीजिए जो बिंदु (–1, –1, 2) से $2\sqrt{2}$ इकाई की दूरी पर है।

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. मान लीजिए एक स्कूल के बारहवीं कक्षा के 30 विद्यार्थियों का समुच्चय A है । माना $f : A \to N$, जहाँ N प्राकृत संख्याओं का समुच्चय है, एक फलन है जिसे $f(x)$ = विद्यार्थी x के रोल नंबर द्वारा प्रदत्त किया गया है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) क्या f एक एकैकी आच्छादक फलन है ? *1*

(ii) (i) के लिए अपने उत्तर के समर्थन में कारण दीजिए। *1*

34. (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

35. (a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

OR

(b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point (–1, –1, 2).

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following :

(i) Is f a bijective function ? *1*

(ii) Give reasons to support your answer to (i). *1*

(iii) (क) मान लीजिए R, जोड़े में विद्यार्थियों के बैठने की व्यवस्था की योजना बनाने के लिए शिक्षक द्वारा परिभाषित एक संबंध है, जो निम्नलिखित द्वारा प्रदत्त है :

$R = \{(x, y) : x, y$ विद्यार्थियों के रोल नंबर हैं, जहाँ $y = 3x\}$

R के अवयवों को सूचीबद्ध कीजिए। क्या संबंध R स्वतुल्य, सममित और संक्रामक है ? अपने उत्तर का औचित्य बताइए। *2*

अथवा

(iii) (ख) मान लीजिए R एक संबंध है जो निम्नलिखित रूप में परिभाषित है :

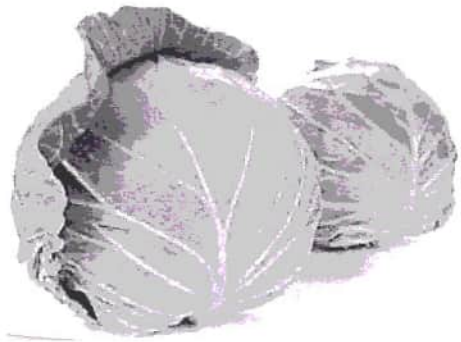
$R = \{(x, y) : x$ और y विद्यार्थियों के रोल नंबर हैं, जहाँ $y = x^3\}$

R के अवयवों को सूचीबद्ध कीजिए। क्या R एक फलन है ? अपने उत्तर का औचित्य बताइए। *2*

प्रकरण अध्ययन – 2

37. एक माली अपने बगीचे में कुछ सब्ज़ियाँ लगाना चाहता है। इसलिए वह बैंगन के पौधों के लिए 10 बीज, पत्तागोभी के पौधों के लिए 12 बीज और मूली के पौधों के लिए 8 बीज खरीदता है। दुकानदार ने उसे बैंगन के अंकुरण, पत्तागोभी के अंकुरण और मूली के अंकुरण की प्रायिकता क्रमश: 25%, 35%, और 40% का आश्वासन दिया। लेकिन इससे पहले कि वह बीज बो पाता, वे एक थैले में मिला दिए गए और उनको यादृच्छया तरीके से बोना पड़ा।

मूली



पत्तागोभी

बैंगन

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) यादृच्छिक रूप से चुने गए एक बीज के अंकुरित होने की प्रायिकता की गणना कीजिए। *2*

(ii) इसकी क्या प्रायिकता है कि यह पत्तागोभी का बीज है, यह जानते हुए कि चुना गया बीज अंकुरित होता है ? *2*

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = 3x\}$.

List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. *2*

OR

(iii) (b) Let R be a relation defined by

$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = x^3\}$.

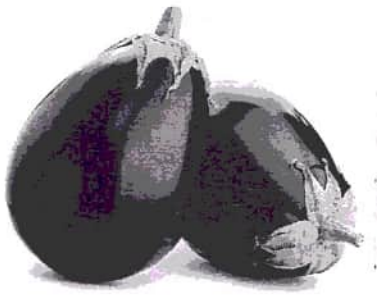
List the elements of R. Is R a function ? Justify your answer. *2*

Case Study – 2

37. A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish

Cabbage



Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate. *2*

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ? *2*

प्रकरण अध्ययन – 3

38. एक बढ़ई को चारों ओर से बंद एक लकड़ी का घनाकार बक्सा बनाना है, जिसका आधार वर्गाकार है और आयतन निश्चित है। क्योंकि उसके पास बॉक्स को पेंट करने के लिए आवश्यक पेंट की कमी है, इसलिए वह चाहता है कि पृष्ठीय क्षेत्रफल न्यूनतम हो।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) लंबाई = चौड़ाई = x मी. तथा ऊँचाई = y मी. लेकर बक्से के पृष्ठीय क्षेत्रफल (S) को x तथा आयतन (V) (जो स्थिर है) के पदों में व्यक्त कीजिए। *1*

(ii) $\frac{dS}{dx}$ ज्ञात कीजिए। *1*

(iii) (क) यदि पृष्ठीय क्षेत्रफल (S) न्यूनतम है, तो x तथा y में संबंध ज्ञात कीजिए। *2*

अथवा

(iii) (ख) यदि पृष्ठीय क्षेत्रफल (S) स्थिर है, तो आयतन (V) = $\frac{1}{4}(Sx - 2x^3)$ है, जहाँ x आधार का एक किनारा है। दर्शाइए कि $x = \sqrt{\frac{S}{6}}$ के लिए आयतन (V) अधिकतम है। *2*

Case Study – 3

38. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. *1*

(ii) Find $\frac{dS}{dx}$. *1*

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum. *2*

OR

(iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. *2*

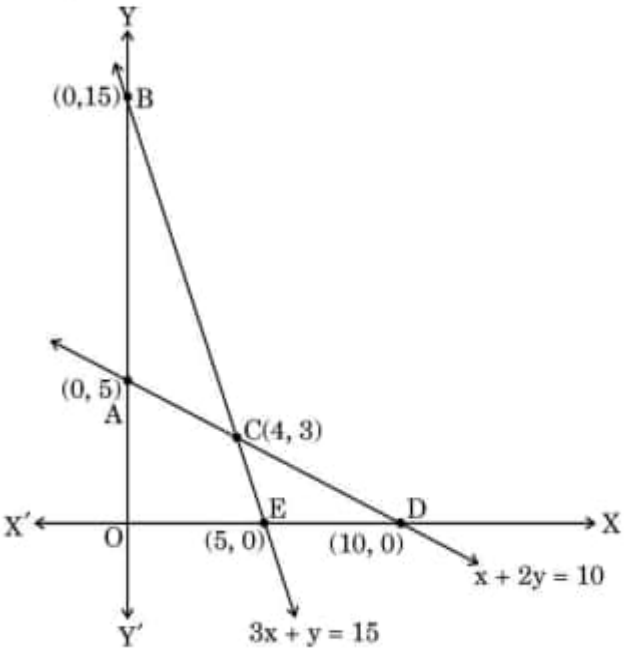
Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Senior Secondary Examination, 2025** **SUBJECT: MATHEMATICS (Q.P. CODE – 65/5/3)**	
General Instructions: -	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

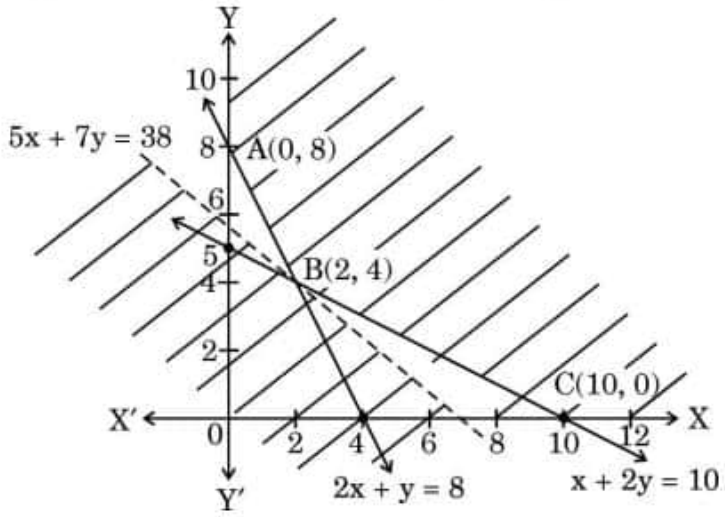
Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A ***This section comprises multiple choice questions (MCQs) of 1 mark each.***	
1.	The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is : (A) $-\frac{\pi}{3}$ (B) $-\frac{2\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$	
Ans	**(D) $\frac{2\pi}{3}$**	**1**
2.	If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $\|A\|$ is : (A) 0 (B) –10 (C) 10 (D) 1	
Ans	**(B) -10**	**1**
3.	If A = kB, where A and B are two square matrices of order n and k is a scalar, then : (A) $\|A\| = k\|B\|$ (B) $\|A\| = k^n\|B\|$ (C) $\|A\| = k + \|B\|$ (D) $\|A\| = \|B\|^k$	
Ans	**(B) $\|A\| = k^n \|B\|$**	**1**
4.	If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at x = 0, then the value of a is : (A) 1 (B) –1 (C) ± 1 (D) 0	
Ans	**(C) ± 1**	**1**

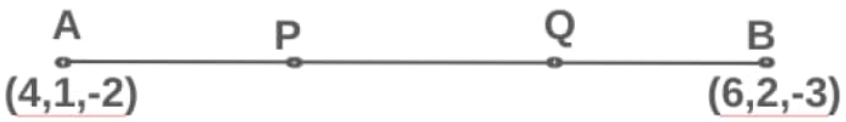
5.	If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is : (A) 3 (B) 7 (C) ± 7 (D) ± 3	
Ans	**(C) ± 7**	**1**
6.	If $P(A \cup B) = 0{\cdot}9$ and $P(A \cap B) = 0{\cdot}4$, then $P(\overline{A}) + P(\overline{B})$ is : (A) 0·3 (B) 1 (C) 1·3 (D) 0·7	
Ans	**(D) 0.7**	**1**
7.	If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is : (A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$ (C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$	
Ans	(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$	**1**
8.	Let A and B be two matrices of suitable orders. Then, which of the following is ***not*** correct ? (A) $(A')' = A$ (B) $(kA)' = kA'$, k is a scalar (C) $(A' + B')' = A + B$ (D) $(AB)' = A'B'$	
Ans	**(D) $(AB)' = A'B'$**	**1**
9.	The area of the region enclosed between the curve $y = x\|x\|$, x-axis, $x = -2$ and $x = 2$ is : (A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) 0 (D) 8	

Ans	**(B)** $\frac{16}{3}$	**1**
10.	If f(x) = {[x], x ∈ R} is the greatest integer function, then the correct statement is : (A) f is continuous but not differentiable at x = 2. (B) f is neither continuous nor differentiable at x = 2. (C) f is continuous as well as differentiable at x = 2. (D) f is not continuous but differentiable at x = 2.	
Ans	**(B) f is neither continuous nor differentiable at x=2.**	**1**
11.	$\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta}\, dx$ is equal to : (A) $2(\sin x + x \cos \theta) + C$ (B) $2(\sin x - x \cos \theta) + C$ (C) $2(\sin x + \sin \theta) + C$ (D) $2(\sin x - x \sin \theta) + C$	
Ans	**(A) 2 (sin x + x cos θ) + C**	1
12.	$\int_0^1 \frac{2x}{5x^2+1}\, dx$ is equal to : (A) $\frac{1}{5}\log 6$ (B) $\frac{1}{5}\log 5$ (C) $\frac{1}{2}\log 6$ (D) $\frac{1}{2}\log 5$	
Ans	**(A)** $\frac{1}{5}$ **log 6**	**1**
13.	The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at : (A) (1, –10) (B) (1, 10) (C) (10, 1) (D) (–10, 1)	
Ans	**(A) (1, -10)**	**1**

14.	The integrating factor of the differential equation $\frac{dx}{dy} = \frac{-(1+\sin x)}{x + y\cos x}$ is : (A) $\log \cos x$ (B) $1 + \sin x$ (C) $e^{(1 + \sin x)}$ (D) $e^{\log \cos x}$	
Ans	**(B) 1 + sin x**	**1**
15.	For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints : $x + 2y \leq 10$ $3x + y \leq 15$ $x, y \geq 0$  The correct feasible region is : (A) ABC (B) AOEC (C) CED (D) Open unbounded region BCD	
Ans	**(B) AOEC**	**1**
16.	The sum of the order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ is : (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) 4	
Ans	**(C) 3**	**1**

17.	The respective values of $\lvert \vec{a} \rvert$ and $\lvert \vec{b} \rvert$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $\lvert \vec{a} \rvert = 3 \lvert \vec{b} \rvert$, are : (A) 48 and 16 (B) 3 and 1 (C) 24 and 8 (D) 6 and 2	
Ans	**(C) 24 and 8**	**1**
18.	Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are : (A) (– 2, – 2) (B) (2, – 2) (C) (– 2, 2) (D) (2, 2)	
Ans	**(C) (−2, 2)**	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	

19.	*Assertion (A) :* The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).  Min Z = 50x + 70y subject to constraints $2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$ Z = 50x + 70y has a minimum value = 380 at B(2, 4). *Reason (R) :* The region representing 50x + 70y < 380 does not have any point common with the feasible region.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
20.	*Assertion (A) :* Let $A = \{x \in R : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function. *Reason (R) :* If $y = -1 \in A$, then $x = \pm \sqrt{-1} \notin A$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
	SECTION-B ***This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.***	
21.	Find the domain of $\sin^{-1}(x^2 - 3)$.	
Ans	**Domain of $\sin^{-1} x$ is $[-1, 1]$** $-1 \leq x^2 - 3 \leq 1 \Rightarrow 2 \leq x^2 \leq 4$ $\Rightarrow$ **Domain** $= [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$	**1** ½ ½

22.	Let the volume of a metallic hollow sphere be constant. If the inner radius increases at the rate of 2 cm/s, find the rate of increase of the outer radius when the radii are 2 cm and 4 cm respectively.	
Ans	$\frac{dr}{dt} = 2$ cm $/s$, , $\left(\frac{dR}{dt}\right)_{R=4,r=2}$ = ?	
	$V = \frac{4}{3}\pi(R^3 - r^3) \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi(3R^2.\frac{dR}{dt} - 3r^2\frac{dr}{dt})$	**1**
	When R = 4 cm and r = 2 cm,	
	$0 = \frac{4}{3}\pi[3(4)^2.\frac{dR}{dt} - 3(2)^2(2)]$	½
	$\Rightarrow \frac{dR}{dt} = \frac{1}{2}$ cm/s	½
23.	A man needs to hang two lanterns on a straight wire whose end points have coordinates A (4, 1, – 2) and B (6, 2, – 3). Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.	
Ans		
	Let P and Q trisect the wire AB.	
	P divides AB in the ratio 1:2 then, coordinate of point P $= \left(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3}\right)$	**1**
	Q divides AB in the ratio 2:1 then, coordinate of point Q $= \left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$	**1**
24.	(a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x. **OR** (b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.	
Ans	**(a) Let** $y = \frac{\sin x}{\sqrt{\cos x}}$	
	$\frac{dy}{dx} = \frac{\sqrt{\cos x}.\cos x - \sin x.\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$	**1½**
	$\Rightarrow \frac{dy}{dx} = \frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{3/2}}$ **or** $\frac{1+\cos^2 x}{2(\cos x)^{3/2}}$	½

	OR	
	(b) $y = 5\cos x - 3\sin x$, **then** $\frac{dy}{dx} = -5.\sin x - 3.\cos x$	**1**
	$\Rightarrow \frac{d^2y}{dx^2} = -5.\cos x + 3.\sin x = -y$	½
	$\Rightarrow \frac{d^2y}{dx^2} + y = 0$	½
25.	(a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ and $4\hat{i} + 3\hat{j} - 2\hat{k}$. **OR** (b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, $\vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.	
Ans	**Let** $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$	
	$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{i} + 10\hat{j} + 17\hat{k}$	½
	$\lvert\vec{a} \times \vec{b}\rvert = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$	½
	Unit vector $\hat{n} = \frac{\vec{a} \times \vec{b}}{\lvert\vec{a} \times \vec{b}\rvert} = \frac{1}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$	½
	$\therefore$ **Required vector** $= \frac{5}{\sqrt{390}}(\hat{i} + 10\hat{j} + 17\hat{k})$	½
	OR	
	(b) $\vec{a}.\vec{b} = \vec{a}.\vec{c} \Rightarrow \vec{a}.(\vec{b} - \vec{c}) = 0$	
	$\Rightarrow$ **either** $\vec{b} = \vec{c}$ **or** $\vec{a} \perp (\vec{b} - \vec{c})$, **since** $\vec{a} \neq 0$	**1**
	Also, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$	
	$\Rightarrow$ **either** $\vec{b} = \vec{c}$ **or** $\vec{a} \parallel (\vec{b} - \vec{c})$, **since** $\vec{a} \neq 0$	½
	Since vectors $\vec{a}$ **and** $(\vec{b} - \vec{c})$ **cannot be** $\parallel$ **and** $\perp$ **simultaneously**	
	Hence $\vec{b} = \vec{c}$	½
	SECTION-C ***This section comprises 6 Short Answer (SA) type questions of 3 marks each.***	
26.	Find the interval/intervals in which the function f(x) = sin 3x – cos 3x, $0 < x < \frac{\pi}{2}$ is strictly increasing.	

Ans	$f'(x) = 3\cos 3x + 3\sin 3x$	**1**
	$f'(x) = 0 \Rightarrow \sin 3x = -\cos 3x \Rightarrow x = \frac{\pi}{4}$	½
	For $x \in \left(0, \frac{\pi}{4}\right)$, $3\cos 3x + 3\sin 3x > 0$	**1**
	$\Rightarrow f'(x) > 0$, f is strictly increasing function in $(0, \frac{\pi}{4})$ or $(0, \frac{\pi}{4}]$	½
27.	(a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 5$, $\lvert\vec{c}\rvert = 7$, then find the angle between $\vec{a}$ and $\vec{b}$. **OR** (b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}\lvert\vec{a} - \vec{b}\rvert = \sin\frac{\theta}{2}$.	
Ans	Given $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \lvert\vec{a} + \vec{b}\rvert = \lvert-\vec{c}\rvert$	**1**
	$\Rightarrow \lvert\vec{a} + \vec{b}\rvert^2 = \lvert\vec{c}\rvert^2 \Rightarrow \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + 2\,\vec{a}.\vec{b} = \lvert\vec{c}\rvert^2$	
	$\Rightarrow 9 + 25 + 2\,\vec{a}.\vec{b} = 49$	**1**
	$\Rightarrow 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = 15$	
	$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$	**1**
	OR	
	(b) $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 1$	½
	$\lvert\vec{a} - \vec{b}\rvert^2 = \lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 - 2\vec{a}.\vec{b}$	**1**
	$= 1 + 1 - 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta$	½
	$= 2 - 2\cos\theta$	
	$= 2\left(2\sin^2\frac{\theta}{2}\right) = 4\sin^2\frac{\theta}{2}$	½
	$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{2}\lvert\vec{a} - \vec{b}\rvert$	½

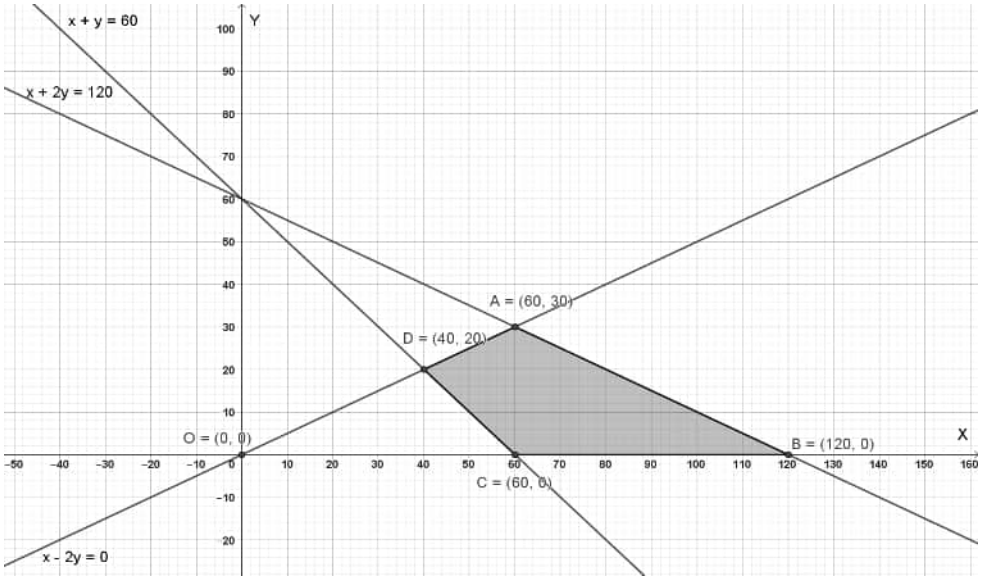
28.	Solve the differential equation $x \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x.$	
Ans	$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$ **Put $y = vx$** $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow v + x\frac{dv}{dx} = v + \sec v$ $\Rightarrow \int \cos v\, dv = \int \frac{dx}{x}$ $\Rightarrow \sin v = \log\lvert x\rvert + c$ $\Rightarrow \sin\frac{y}{x} = \log\lvert x\rvert + c$	½ ½ 1 ½ ½
29.	(a) The probability that a student buys a colouring book is 0·7 and that she buys a box of colours is 0·2. The probability that she buys a colouring book, given that she buys a box of colours, is 0·3. Find the probability that the student : (i) Buys both the colouring book and the box of colours. (ii) Buys a box of colours given that she buys the colouring book. **OR** (b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find : (i) The probability distribution of the number of oranges he draws. (ii) The expectation of the random variable (number of oranges).	
Ans	**(a) Let A be the event of buying colouring book and B be the event of buying coloured box.**	½

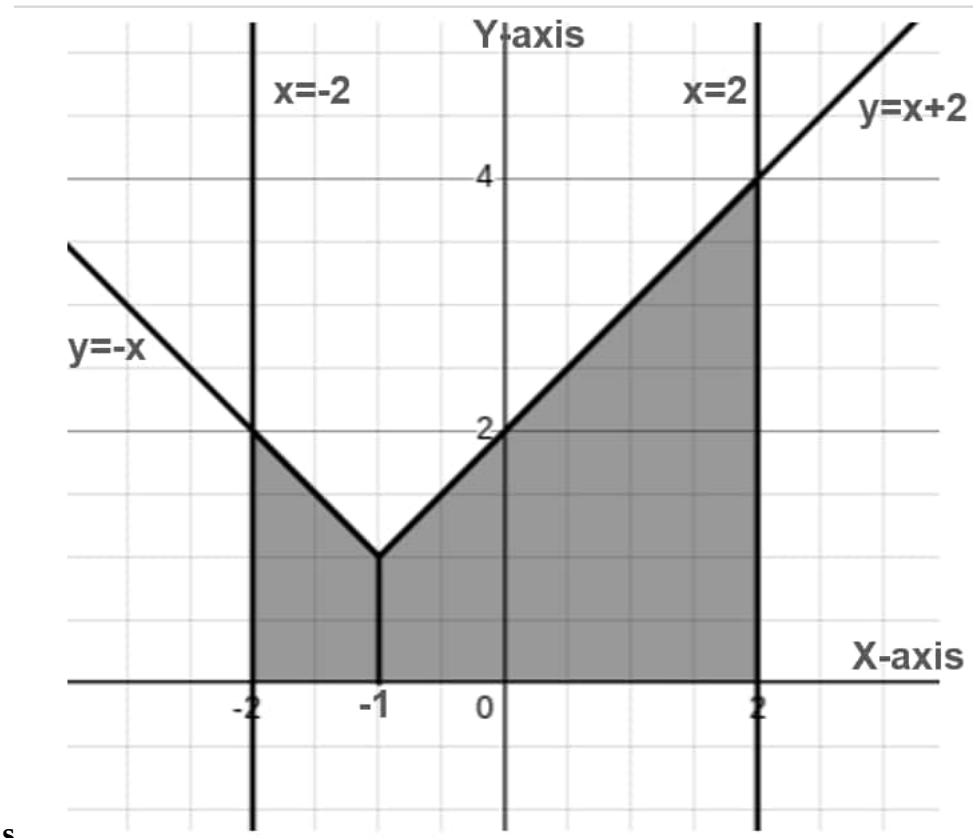
	$P(A) = 0.7, \quad P(B) = 0.2, \quad P(A/B) = 0.3$	½
	(i) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow 0.3 = \frac{P(A \cap B)}{0.2}$	
	$\Rightarrow P(A \cap B) = 0.06$ **or** $\frac{3}{50}$	**1**
	(ii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$	
	$= \frac{0.06}{0.7} = \frac{3}{35}$ **or** 0.086	**1**
	OR	
	(b) Let X be random variable for number of oranges.	
	X = 0, 1, 2, 3	½
	Let A be the event that orange is drawn.	
	$P(A) = \frac{4}{10} = \frac{2}{5}, \quad P(\bar{A}) = 1 - \frac{2}{5} = \frac{3}{5}$	½
	(i)	

X	0	1	2	3
P(X)	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

1

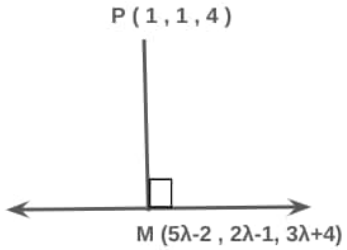
	(ii) $E(X) = \sum p_i x_i = 0 \times \frac{27}{125} + 1 \times \frac{54}{125} + 2 \times \frac{36}{125} + 3 \times \frac{8}{125}$	½
	$= \frac{150}{125}$ **or** $\frac{6}{5}$	½
30.	(a) Find : $\int \frac{2x}{(x^2+3)(x^2-5)}\,dx$	
	OR	
	(b) Evaluate : $\int_1^4 \left(\lvert x-2\rvert + \lvert x-4\rvert\right)\,dx$	
Ans	**(a)** Let $I = \int \frac{2x}{(x^2+3)(x^2-5)}\,dx$	
	Put $x^2 = t \Rightarrow 2x.\,dx = dt$	½

	$\Rightarrow I = \int \frac{dt}{(t+3)(t-5)}$ $= \int \left(-\frac{1}{8(t+3)} + \frac{1}{8(t-5)}\right) dt$ $= \frac{1}{8}[log\|t-5| - \log|t+3|] + c$ $= \frac{1}{8}\log\left\|\frac{x^2-5}{x^2+3}\right\| + c$ **OR** **(b)** $\int_1^4 (\|x-2| + |x-4|)\, dx$ $= \int_1^2 (2-x)\, dx + \int_2^4 (x-2) dx - \int_1^4 (x-4)\, dx$ $= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-4)^2}{2}\right]_1^4$ $= \frac{1}{2} + 2 + \frac{9}{2} = 7$	1 1 ½ 1½ 1 ½
31.	In the Linear Programming Problem (LPP), find the point/points giving maximum value for Z = 5x + 10y subject to constraints $x + 2y \leq 120$ $x + y \geq 60$ $x - 2y \geq 0$ $x, y \geq 0$	
Ans		**1½ for correct graph and correct feasible region**

	<table><tr><th>Corner Points</th><th>Value of Z</th></tr><tr><td>A (60, 30)</td><td>600</td></tr><tr><td>B (120, 0)</td><td>600</td></tr><tr><td>C (60, 0)</td><td>300</td></tr><tr><td>D (40, 20)</td><td>400</td></tr></table> **Since Z is maximum on points A and B** **Hence all points lying on segment AB give maximum Z.**	**1** ½
	SECTION-D ***This section comprises 4 Long Answer (LA) type questions of 5 marks each.***	
32.	In a rough sketch, mark the region bounded by $y = 1 + \|x + 1\|$, $x = -2$, $x = 2$ and $y = 0$. Using integration, find the area of the marked region.	
Ans **s**		**2 marks for correct graph and shading**

	Required area $= \int_{-2}^{-1}(-x)\ dx + \int_{-1}^{2}(x+2)\,dx$ $= -\frac{1}{2}[x^2]_{-2}^{-1} + \left[\frac{1}{2}x^2 + 2x\right]_{-1}^{2}$ $= 9$	**1½** **1** **½**
33.	Three students run on a racing track such that their speeds add up to 6 km/h. However, double the speed of the third runner added to the speed of the first results in 7 km/h. If thrice the speed of the first runner is added to the original speeds of the other two, the result is 12 km/h. Using matrix method, find the original speed of each runner.	
Ans	**Let original speed of three runners be x, y and z respectively.** **Then x + y + z = 6 ; x + 2z = 7 ; 3x + y + z = 12** **Let** $A = \begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}$, $X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B = \begin{bmatrix}6\\ 7\\ 12\end{bmatrix}$ $|A| = 4 \neq 0 \implies A^{-1}$ **exists** $AX = B \implies X = A^{-1}B$ $\text{adj}(A) = \begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}$ $A^{-1} = \frac{1}{4}\begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}$ $\therefore \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{4}\begin{bmatrix}-2 & 0 & 2\\ 5 & -2 & -1\\ 1 & 2 & -1\end{bmatrix}\begin{bmatrix}6\\ 7\\ 12\end{bmatrix} = \begin{bmatrix}3\\ 1\\ 2\end{bmatrix}$ **Hence the original speed of three runners are 3km/h, 1km/h and 2 km/h respectively.**	**1½** **½** **½** **1½** **1**

34.	(a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number. **OR** (b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.	
Ans	**(a) Let** $u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}}.\log a.\left(1-\frac{1}{t^2}\right)$	**2**
	$v = \left(t+\frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}.\left(1-\frac{1}{t^2}\right)$	**2**
	$\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$	**1**
	OR	
	(b) Let $u = y^x$, $v = x^y$ **and** $w = x^x$	
	$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ .....(i)	**1**
	$u = y^x \Rightarrow \log u = x.\log y \Rightarrow \frac{1}{u}.\frac{du}{dx} = \frac{x}{y}.\frac{dy}{dx} + \log y$	
	$\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y}.\frac{dy}{dx} + \log y\right) = xy^{x-1}\frac{dy}{dx} + y^x \log y$	**1**
	$v = x^y \Rightarrow \log v = y.\log x \Rightarrow \frac{1}{v}.\frac{dv}{dx} = \frac{y}{x} + \log x.\frac{dy}{dx}$	
	$\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x.\frac{dy}{dx}\right) = yx^{y-1} + x^y \log x \frac{dy}{dx}$	**1**
	$w = x^x \Rightarrow \log w = x.\log x \Rightarrow \frac{1}{w}.\frac{dw}{dx} = 1 + \log x$	
	$\Rightarrow \frac{dw}{dx} = x^x.(1+\log x)$	**1**
	∴ From (i), we get	
	$xy^{x-1}.\frac{dy}{dx} + y^x.\log y + yx^{y-1} + x^y.\log x.\frac{dy}{dx} + x^x.(1+\log x) = 0$	
	$\Rightarrow \frac{dy}{dx} = -\frac{x^x.(1+\log x) + y^x.\log y + yx^{y-1}}{x.y^{x-1} + x^y.\log x}$	**1**

35.	(a) Find the foot of the perpendicular drawn from the point (1, 1, 4) on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$. **OR** (b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point $(-1, -1, 2)$.	
Ans	**(a) Let** $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ **Coordinate of general point on the given line are** $M\ (5\lambda - 2, 2\lambda - 1, 3\lambda + 4)$  **Direction Ratios of PM vector are** $< 5\lambda - 3, 2\lambda - 2, 3\lambda >$ **Since, PM** $\perp l$ $\Rightarrow 5(5\lambda - 3) + 2(2\lambda - 2) + 3(3\lambda) = 0$ $\Rightarrow \lambda = \frac{1}{2}$ **Hence, coordinates of M are** $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$ **OR** **(b) Equation of given line be** $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ **(say)** **Coordinate of any general point on the line are** $P\ (3\lambda + 1, 2\lambda - 1, 3\lambda + 4)$. **Let distance of point P from** $(-1, -1, 2)$ **is** $2\sqrt{2}$. $\Rightarrow \sqrt{(3\lambda + 2)^2 + (2\lambda)^2 + (3\lambda + 2)^2} = 2\sqrt{2}$ $\Rightarrow 22\lambda^2 + 24\lambda = 0$ $\Rightarrow \lambda = 0$ **or** $\lambda = -\frac{12}{11}$	**1** **1** **1** **1** **1** **1** **1½** **1**

	Hence, coordinates of point P are $(1, -1, 4)$ or $\left(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11}\right)$	**1½**
	SECTION-E ***This section comprises 3 case study-based questions of 4 marks each***	
36.	**Case Study – 1** Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x) = Roll Number of student x. On the basis of the given information, answer the following : (i) Is f a bijective function ? *1* (ii) Give reasons to support your answer to (i). *1* (iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer. *2* **OR** (iii) (b) Let R be a relation defined by R = {(x, y) : x, y are Roll Numbers of students such that $y = x^3$}. List the elements of R. Is R a function ? Justify your answer. *2*	
Ans	**(i) No, f is not bijective function**	**1**
	(ii) Range = $\{1, 2, 3, 4, \ldots\ldots\ldots\ldots\ldots, 30\}$ and codomain= N	**½**
	Since, Range ≠ codomain ⇒ f is not onto and hence f is not bijective.	**½**
	(iii) (a)	
	$R = \{(1,3), (2,6), (3,9), (4,12), (5,15), (6,18), (7,21), (8,24), (9,27), (10,30)\}$	**1**
	Since $(1,1) \notin R$ ⟹ R is not reflexive. **$(1,3) \in R$ but $(3,1) \notin R$ ⟹ R is not symmetric** **$(1,3) \in R, (3,9) \in R$ but $(1,9) \notin R$ ⟹ R is not transitive.**	**1**
	OR	
	(iii) (b) $R = \{(1,1), (2,8), (3,27)\}$	**1**
	∵ elements 4, 5, 6 … 30 do not have an image. Hence the above relation is not a function.	**1**

37.

Case Study – 2

A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Radish

Cabbage

Brinjal

Based upon the above information, answer the following questions :

(i) Calculate the probability of a randomly chosen seed to germinate. *2*

(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates ? *2*

Ans

Let A: Event that chosen seed germinates.

B: Event that Brinjal seed is chosen.

C: Event that Cabbage seed is chosen.

R: Event that Radish seed is chosen.

$P(B) = \frac{10}{30}$; $P(C) = \frac{12}{30}$; $P(R) = \frac{8}{30}$;

$P\left(\frac{A}{B}\right) = \frac{25}{100}$; $P\left(\frac{A}{C}\right) = \frac{35}{100}$; $P\left(\frac{A}{R}\right) = \frac{40}{100}$ **1**

(i) $P(A) = P(B).P\left(\frac{A}{B}\right) + P(C).P\left(\frac{A}{C}\right) + P(R).P\left(\frac{A}{R}\right)$

$= \frac{10}{30} \times \frac{25}{100} + \frac{12}{30} \times \frac{35}{100} + \frac{8}{30} \times \frac{40}{100}$

$= \frac{990}{3000}$ **or** $\frac{33}{100}$ **1**

(ii) **(a)** $P\left(\frac{C}{A}\right) = \frac{P(C).P\left(\frac{A}{C}\right)}{P(B).P\left(\frac{A}{B}\right)+P(C).P\left(\frac{A}{C}\right)+P(R).P\left(\frac{A}{R}\right)}$

$= \frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$ **1**

$= \frac{42}{99}$ **or** $\frac{14}{33}$ **1**

38.

Case Study – 3

A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant. *1*

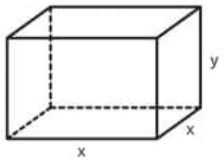
(ii) Find $\frac{dS}{dx}$. *1*

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum. *2*

OR

(iii) (b) If surface area (S) is constant, the volume (V) = $\frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$. *2*

Ans



(i) $V = x^2y \Rightarrow y = \frac{V}{x^2} \ldots\ldots\ldots\ldots (i)$

Hence, $S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}$ **1**

(ii) $\frac{dS}{dx} = 4\left(x - \frac{V}{x^2}\right)$ **1**

(iii) **(a)** $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2y = x^3 \Rightarrow y = x$ **1**

$\frac{d^2S}{dx^2} = 4\left(1 + \frac{2V}{x^3}\right) = 12 > 0 \Rightarrow$ **S is minimum if** $y = x$. **1**

OR

(iii) **(b)** $V = \frac{1}{4}(Sx - 2x^3) \Rightarrow \frac{dV}{dx} = \frac{1}{4}(S - 6x^2)$ **1**

Put $\frac{dV}{dx} = 0 \Rightarrow x = \sqrt{\frac{S}{6}}$ ½

$\left(\frac{d^2V}{dx^2}\right)_{x=\sqrt{\frac{S}{6}}} = -3\sqrt{\frac{S}{6}} < 0 \Rightarrow$ **Volume is maximum for** $x = \sqrt{\frac{S}{6}}$. ½