



Series WX1YZ/5



SET~1

प्रश्न-पत्र कोड
Q.P. Code

30/5/1

रोल नं.

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित (मानक)

MATHEMATICS (STANDARD)

*

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

नोट / NOTE :

- (i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 27 हैं ।
Please check that this question paper contains 27 printed pages.
- (ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।
Please check that this question paper contains 38 questions.
- (iv) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।
Please write down the serial number of the question in the answer-book before attempting it.
- (v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.



General Instructions :

Read the following instructions carefully and follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each. Internal choice is provided in **2** marks questions in each case-study.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.
- (ix) Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
- (x) Use of calculators is **not** allowed.

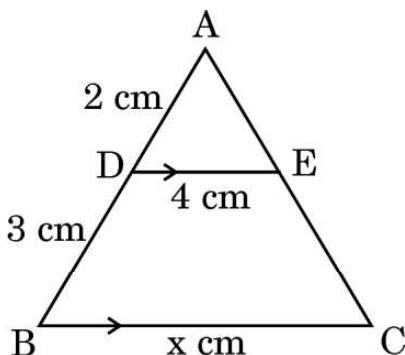
SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The number of polynomials having zeroes –3 and 5 is :
 - (a) only one
 - (b) infinite
 - (c) exactly two
 - (d) at most two
2. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if :
 - (a) $a = b$
 - (b) $3a = 2b$
 - (c) $2a = 3b$
 - (d) $ab = 6$
3. The common difference of the A.P. whose n^{th} term is given by $a_n = 3n + 7$, is :
 - (a) 7
 - (b) 3
 - (c) $3n$
 - (d) 1



4. In the given figure, $DE \parallel BC$. The value of x is :



(a) 6 (b) 12.5
(c) 8 (d) 10

5. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is :

(a) $x^2 - 4x + 1 = 0$ (b) $x^2 + 4x + 1 = 0$
(c) $4x^2 - 3 = 0$ (d) $x^2 - 1 = 0$

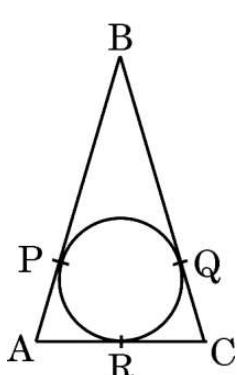
6. If $\tan \theta = \frac{5}{12}$, then the value of $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$ is :

(a) $-\frac{17}{7}$ (b) $\frac{17}{7}$
(c) $\frac{17}{13}$ (d) $-\frac{7}{13}$

7. The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is :

(a) 6 units (b) 4 units
(c) 2 units (d) 3 units

8. In the given figure, $AB = BC = 10$ cm. If $AC = 7$ cm, then the length of BP is :



(a) 3.5 cm (b) 7 cm
(c) 6.5 cm (d) 5 cm



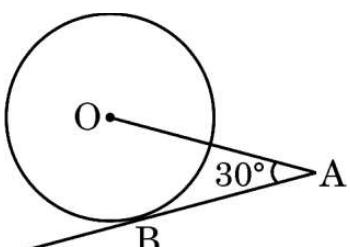
9. Water in a river which is 3 m deep and 40 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in 2 minutes ?

(a) 800 m^3 (b) 4000 m^3
(c) 8000 m^3 (d) 2000 m^3

10. If the mean and the median of a data are 12 and 15 respectively, then its mode is :

(a) 13.5 (b) 21
(c) 6 (d) 14

11. In the given figure, AB is a tangent to the circle centered at O. If $OA = 6 \text{ cm}$ and $\angle OAB = 30^\circ$, then the radius of the circle is :



(a) 3 cm (b) $3\sqrt{3} \text{ cm}$
(c) 2 cm (d) $\sqrt{3} \text{ cm}$

12. $\left(\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} \right)$ is equal to :

(a) $\sin 60^\circ$ (b) $\cos 60^\circ$
(c) $\tan 60^\circ$ (d) $\sin 30^\circ$

13. In $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$. Which of the following makes the two triangles similar ?

(a) $\angle A = \angle D$ (b) $\angle B = \angle D$
(c) $\angle B = \angle E$ (d) $\angle A = \angle F$



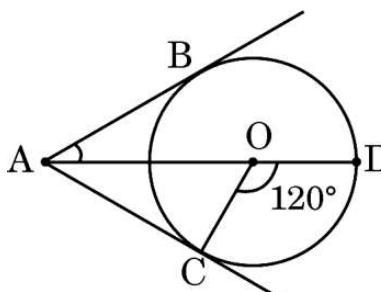
14. The 11th term from the end of the A.P. : 10, 7, 4,, -62 is :

(a) 25 (b) 16
(c) -32 (d) 0

15. Two coins are tossed together. The probability of getting at least one tail is :

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) 1

16. In the given figure, AC and AB are tangents to a circle centered at O. If $\angle COD = 120^\circ$, then $\angle BAO$ is equal to :



(a) 30° (b) 60°
(c) 45° (d) 90°

17. Which of the following numbers **cannot** be the probability of happening of an event ?

(a) 0 (b) $\frac{7}{0.01}$
(c) 0.07 (d) $\frac{0.07}{3}$

18. If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data :

(a) decreases by 2
(b) remains unchanged
(c) decreases by $2n$
(d) decreases by 1



Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (c) Assertion (A) is true, but Reason (R) is false.
- (d) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : If the points A(4, 3) and B(x, 5) lie on a circle with centre O(2, 3), then the value of x is 2.

Reason (R) : Centre of a circle is the mid-point of each chord of the circle.

20. Assertion (A) : The number 5^n cannot end with the digit 0, where n is a natural number.

Reason (R) : Prime factorisation of 5 has only two factors, 1 and 5.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

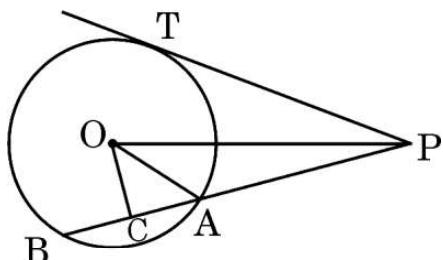
21. (a) The line segment joining the points A(4, - 5) and B(4, 5) is divided by the point P such that $AP : AB = 2 : 5$. Find the coordinates of P.

OR

(b) Point P(x, y) is equidistant from points A(5, 1) and B(1, 5). Prove that $x = y$.



22. In the given figure, PT is a tangent to the circle centered at O. OC is perpendicular to chord AB. Prove that $PA \cdot PB = PC^2 - AC^2$.



23. Using prime factorisation, find HCF and LCM of 96 and 120.

24. Find the ratio in which y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$.

25. (a) If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

OR

(b) Prove that :

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Prove that $\sqrt{3}$ is an irrational number.

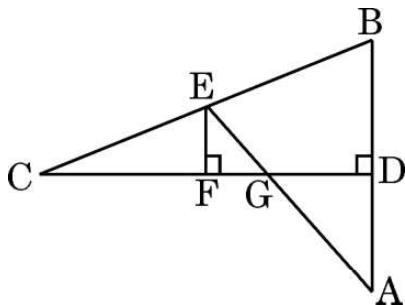
OR

(b) The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next ?



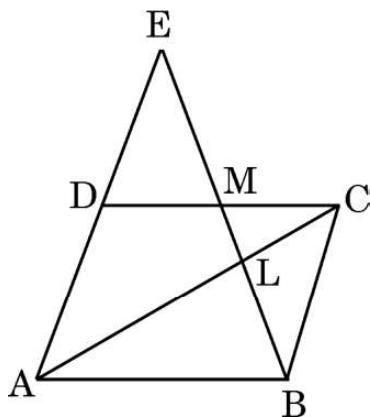
27. If p^{th} term of an A.P. is q and q^{th} term is p , then prove that its n^{th} term is $(p + q - n)$.

28. (a) In the given figure, CD is the perpendicular bisector of AB . EF is perpendicular to CD . AE intersects CD at G . Prove that $\frac{CF}{CD} = \frac{FG}{DG}$.



OR

(b) In the given figure, $ABCD$ is a parallelogram. BE bisects CD at M and intersects AC at L . Prove that $EL = 2BL$.



29. Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds.

30. Prove that :

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \cosec \theta$$



31. Find the mean of the following frequency distribution :

Classes	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
Frequency	14	22	16	6	5	3	4

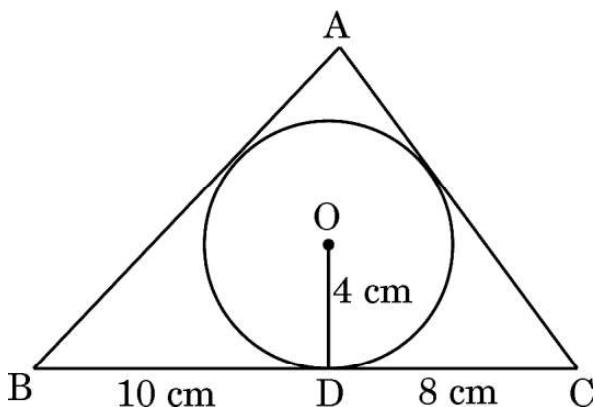
SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60° , while another observer 100 m away estimates the angle of elevation to be 30° . Find :

- The height of the basket from the ground.
- The distance of the basket from the first observer's eye.
- The horizontal distance of the second observer from the basket.

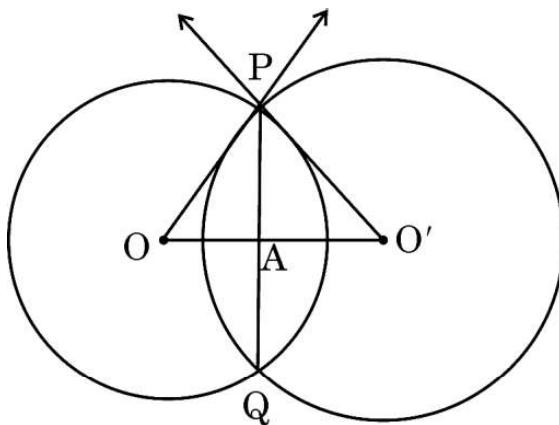
33. (a) A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides AB and AC, if it is given that area $\Delta ABC = 90 \text{ cm}^2$.



OR



(b) Two circles with centres O and O' of radii 6 cm and 8 cm, respectively intersect at two points P and Q such that OP and $O'P$ are tangents to the two circles. Find the length of the common chord PQ .



34. (a) A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed ?

OR

(b) Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.

35. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use $\pi = 3.14$)

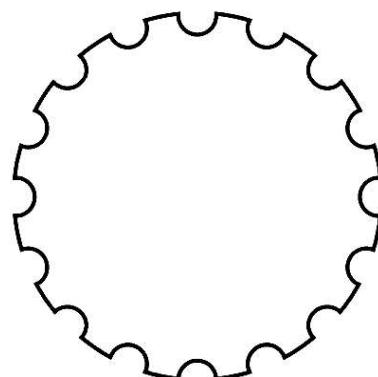


SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A golf ball is spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.



Based on the above, answer the following questions :

(i) Find the surface area of one such dimple. 1

(ii) Find the volume of the material dug out to make one dimple. 1

(iii) (a) Find the total surface area exposed to the surroundings. 2

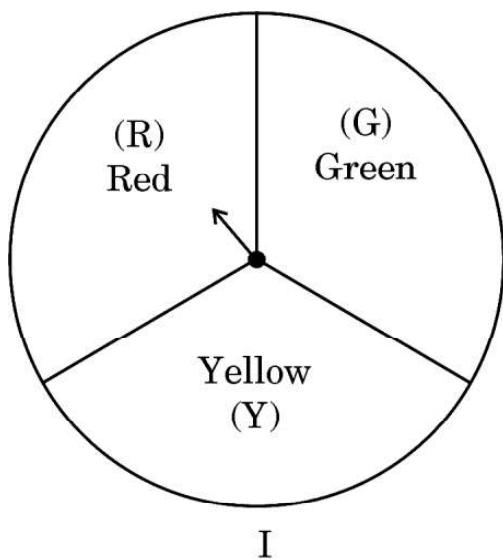
OR

(iii) (b) Find the volume of the golf ball. 2

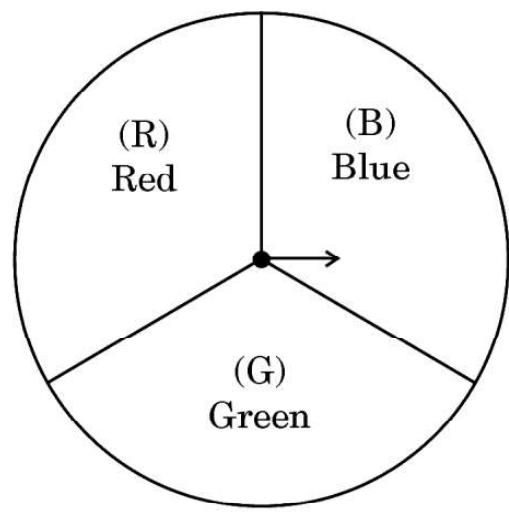


Case Study – 2

37. A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.



I



II

Making Purple : Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you ‘win’. One such outcome is written as ‘RB’.

Based on the above, answer the following questions :

(i) List all possible outcomes of the game. 1

(ii) Find the probability of ‘Making Purple’. 1

(iii) (a) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school.
If 99 participants played, calculate how much fund could the school have collected. 2

OR

(iii) (b) If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school ? (Number of participants = 99) 2



Case Study – 3

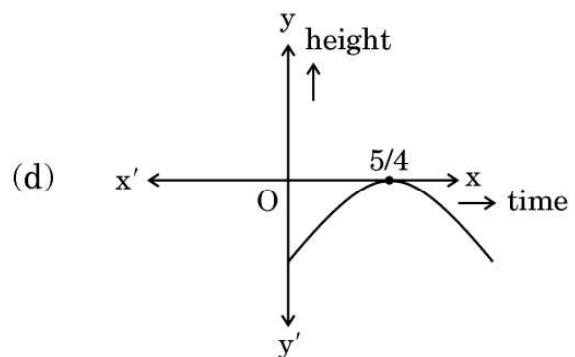
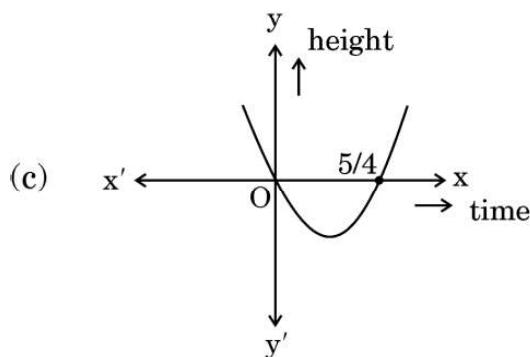
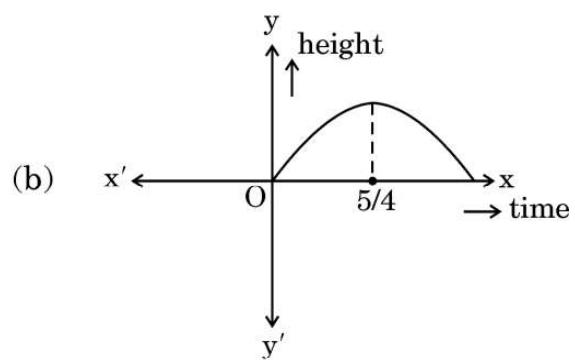
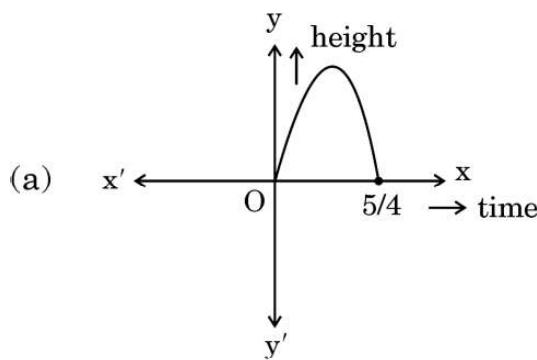
38. In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.



Based on the above, answer the following questions :

(i) Find zeroes of polynomial $p(t) = 20t - 16t^2$. 1

(ii) Which of the following types of graph represents $p(t)$? 1





(iii) (a) What would be the value of h at $t = \frac{3}{2}$? Interpret the result. 2

OR

(iii) (b) How much distance has the dolphin covered before hitting the water level again ? 2

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Secondary School Examination, 2023
MATHEMATICS PAPER CODE 30/5/1

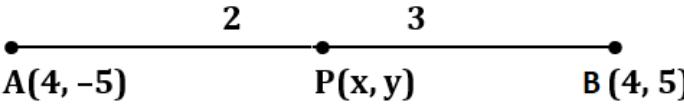
General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC.”
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.
4	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS ‘X’ be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note “Extra Question”.</u>
10	<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note “Extra Question”.</u>
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	<p>Ensure that you do not make the following common types of errors committed by the Examiner in the past: -</p> <ul style="list-style-type: none"> ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the “ Guidelines for spot Evaluation ” before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

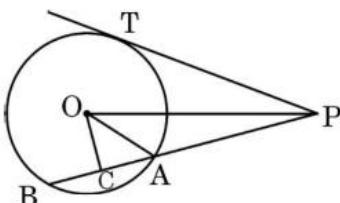
**MARKING SCHEME
MATHEMATICS (Subject Code-041)
(PAPER CODE: 30/5/1)**

18.	<p>If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data :</p> <p>(a) decreases by 2 (b) remains unchanged (c) decreases by $2n$ (d) decreases by 1</p>	
Sol.	(a) decreases by 2	1
	<p><i>Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled as Assertion (A) and the other is labelled as Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.</i></p> <p>(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.</p>	
19.	<p><i>Assertion (A) : If the points A(4, 3) and B(x, 5) lie on a circle with centre O(2, 3), then the value of x is 2.</i></p> <p><i>Reason (R) : Centre of a circle is the mid-point of each chord of the circle.</i></p>	
Sol.	(c) Assertion (A) is true, but Reason (R) is false	1
20.	<p><i>Assertion (A) : The number 5^n cannot end with the digit 0, where n is a natural number.</i></p> <p><i>Reason (R) : Prime factorisation of 5 has only two factors, 1 and 5.</i></p>	

Sol.	(c) Assertion (A) is true, but Reason (R) is false	1
	SECTION B This section comprises of very short answer (VSA) type questions of 2 marks each.	
21(a).	The line segment joining the points A(4, - 5) and B(4, 5) is divided by the point P such that $AP : AB = 2 : 5$. Find the coordinates of P.	
Sol.	$AP : AB = 2 : 5 \Rightarrow AP : PB = 2 : 3$  $x = \frac{8 + 12}{5} = 4, y = \frac{10 - 15}{5} = -1$ Point P is (4, -1)	$\frac{1}{2}$ $\frac{1}{2}$
	OR	
21(b).	Point P(x, y) is equidistant from points A(5, 1) and B(1, 5). Prove that $x = y$.	
Sol.	$PA^2 = PB^2 \Rightarrow (x - 5)^2 + (y - 1)^2 = (x - 1)^2 + (y - 5)^2$ $\Rightarrow x = y$	$\frac{1}{2}$

22.

In the given figure, PT is a tangent to the circle centered at O. OC is perpendicular to chord AB. Prove that $PA \cdot PB = PC^2 - AC^2$.



Sol.

$$\begin{aligned}
 PA \cdot PB &= (PC - AC) \cdot (PC + BC) \\
 &= (PC - AC)(PC + AC) \quad [AC = BC] \\
 &= PC^2 - AC^2
 \end{aligned}$$

1
 $\frac{1}{2}$
 $\frac{1}{2}$

23.

Using prime factorisation, find HCF and LCM of 96 and 120.

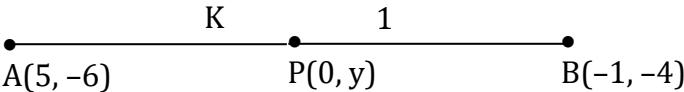
ol.

$$\begin{aligned}
 96 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \\
 &= 2^5 \times 3 \\
 120 &= 2 \times 2 \times 2 \times 3 \times 5 \\
 &= 2^3 \times 3 \times 5 \\
 \text{HCF} &= 24 \\
 \text{LCM} &= 480
 \end{aligned}$$

1
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$

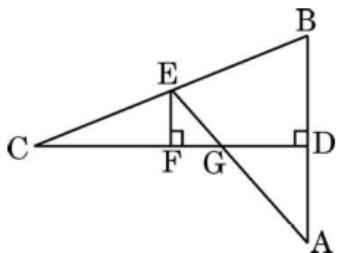
24.

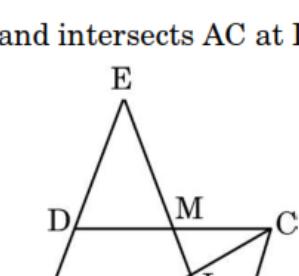
Find the ratio in which y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$.

Sol.	 <p>Let the point of division be $P(0, y)$ which divides AB in the ratio $K : 1$</p> $0 = \frac{-K + 5}{K + 1} \Rightarrow K = 5$ <p>Ratio is 5:1</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$
25(a).	<p>If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.</p>	
Sol.	$ \begin{aligned} m^2 + n^2 &= (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 \\ &= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) \\ &= a^2 + b^2 \end{aligned} $	$\frac{1}{2}$ 1 $\frac{1}{2}$
	<p>OR</p>	
25(b).	<p>Prove that :</p> $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$	
Sol.	$ \begin{aligned} \text{LHS} &= \frac{\sqrt{\sec A - 1}}{\sqrt{\sec A + 1}} + \frac{\sqrt{\sec A + 1}}{\sqrt{\sec A - 1}} \\ &= \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec^2 A - 1}} \end{aligned} $	1

	$= \frac{2 \sec A}{\tan A}$ $= 2 \operatorname{cosec} A = \text{RHS}$	$\frac{1}{2}$ $\frac{1}{2}$
	SECTION C This section comprises of short answer (SA) type questions of 3 marks each.	
26(a).	Prove that $\sqrt{3}$ is an irrational number.	
Sol.	<p>Let $\sqrt{3}$ be a rational number.</p> $\therefore \sqrt{3} = \frac{p}{q}, \text{ let } p \text{ & } q \text{ be co-primes and } q \neq 0$ $3q^2 = p^2 \Rightarrow p^2 \text{ is divisible by 3} \Rightarrow p \text{ is divisible by 3}$ $\Rightarrow p = 3a, \text{ where 'a' is some integer} \quad \text{----- (i)}$ $9a^2 = 3q^2 \Rightarrow q^2 = 3a^2 \Rightarrow q^2 \text{ is divisible by 3} \Rightarrow q \text{ is divisible by 3}$ $\Rightarrow q = 3b, \text{ where 'b' is some integer} \quad \text{----- (ii)}$ <p>(i) and (ii) leads to contradiction as 'p' and 'q' are co-primes.</p> $\therefore \sqrt{3} \text{ is an irrational number.}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
	OR	
26(b)	The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next ?	

Sol.	<p>$LCM = 432$</p> <p>i.e. $\frac{432}{60} = 7 \text{ min } 12 \text{ sec.}$</p> <p>$\Rightarrow$ traffic lights will change simultaneously again at 7 : 7 : 12 a.m.</p>	2 1
27.	If p^{th} term of an A.P. is q and q^{th} term is p , then prove that its n^{th} term is $(p + q - n)$.	
Sol.	$a_p = a + (p - 1)d = q \quad \text{(i)}$ $a_q = a + (q - 1)d = p \quad \text{(ii)}$ <p>Solving (i) and (ii)</p> $d = -1, a = q + p - 1$ $a_n = (q + p - 1) + (n - 1)(-1) = q + p - n$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ 1
28(a).	In the given figure, CD is the perpendicular bisector of AB . EF is perpendicular to CD . AE intersects CD at G . Prove that $\frac{CF}{CD} = \frac{FG}{DG}$.	

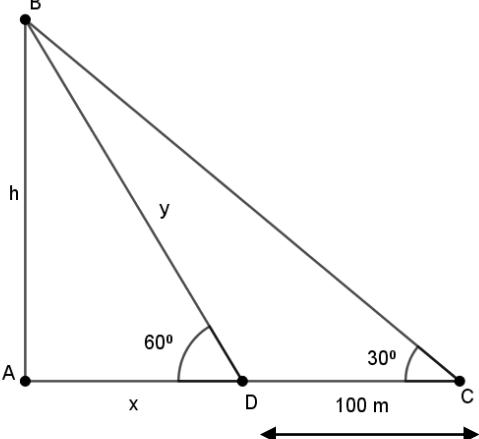


Sol.	$\Delta EFG \sim \Delta ADG$ $\Rightarrow \frac{EF}{AD} = \frac{FG}{DG} \quad \text{(i)}$	1
	$\Delta EFC \sim \Delta BDC$ $\Rightarrow \frac{EF}{BD} = \frac{CF}{CD} \quad \text{(ii)}$	1
	$\Rightarrow \frac{EF}{AD} = \frac{CF}{CD} \quad \{BD = AD\} \quad \text{(iii)}$	$\frac{1}{2}$
	Using (i) and (ii)	
	$\frac{FG}{DG} = \frac{CF}{CD}$	$\frac{1}{2}$
	OR	
28(b).	In the given figure, ABCD is a parallelogram. BE bisects CD at M and intersects AC at L. Prove that $EL = 2BL$.	
		

Sol.	<p>$\Delta ALE \sim \Delta CLB$</p> $\Rightarrow \frac{AL}{CL} = \frac{EL}{BL} \quad \text{(i)}$	1
	<p>Also $\Delta CLM \sim \Delta ALB$</p>	
	$\Rightarrow \frac{AL}{CL} = \frac{AB}{CM}$	1
	$\Rightarrow \frac{AL}{CL} = \frac{CD}{CM} \quad \{AB = CD\} \quad \text{(ii)}$	1/2
	<p>Using (i) and (ii)</p>	
	$\frac{EL}{BL} = \frac{2CM}{CM}$	
	$\Rightarrow EL = 2BL$	1/2
29.	<p>Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds.</p>	
Sol.	<p>Let walking speeds be x km/hr. and y km/hr. ($x > y$)</p>	
	<p>ATQ, $2x + 2y = 16$</p>	1
	<p>and $8x - 8y = 16$</p>	1
	<p>Solving to get $x = 5$, $y = 3$</p>	$\frac{1}{2} + \frac{1}{2}$
	<p>Speeds are 5 km/hr. 3 km/hr.</p>	

30.	<p>Prove that :</p> $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \cosec \theta$																	
Sol.	$\begin{aligned} \text{LHS} &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta(\cos \theta - \sin \theta)} \\ &= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta(\sin \theta - \cos \theta)} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} + 1 \\ &= 1 + \cosec \theta \sec \theta = \text{RHS} \end{aligned}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$																
31.	<p>Find the mean of the following frequency distribution :</p> <table border="1" data-bbox="174 1100 1123 1229"> <thead> <tr> <th>Classes</th><th>25 – 30</th><th>30 – 35</th><th>35 – 40</th><th>40 – 45</th><th>45 – 50</th><th>50 – 55</th><th>55 – 60</th></tr> </thead> <tbody> <tr> <th>Frequency</th><td>14</td><td>22</td><td>16</td><td>6</td><td>5</td><td>3</td><td>4</td></tr> </tbody> </table>	Classes	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	Frequency	14	22	16	6	5	3	4	
Classes	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60											
Frequency	14	22	16	6	5	3	4											

Sol.	<table border="1" data-bbox="174 1257 956 1699"> <thead> <tr> <th>C.I.</th><th>x</th><th>f</th><th>$u = \frac{x - 42.5}{5}$</th><th>fu</th></tr> </thead> <tbody> <tr> <td>25 – 30</td><td>27.5</td><td>14</td><td>-3</td><td>-42</td></tr> <tr> <td>30 – 35</td><td>32.5</td><td>22</td><td>-2</td><td>-44</td></tr> <tr> <td>35 – 40</td><td>37.5</td><td>16</td><td>-1</td><td>-16</td></tr> <tr> <td>40 – 45</td><td>42.5</td><td>6</td><td>0</td><td>0</td></tr> <tr> <td>45 – 50</td><td>47.5</td><td>5</td><td>1</td><td>5</td></tr> <tr> <td>50 – 55</td><td>52.5</td><td>3</td><td>2</td><td>6</td></tr> <tr> <td>55 – 60</td><td>57.5</td><td>4</td><td>3</td><td>12</td></tr> <tr> <td></td><td></td><td>70</td><td></td><td>-79</td></tr> </tbody> </table>	C.I.	x	f	$u = \frac{x - 42.5}{5}$	fu	25 – 30	27.5	14	-3	-42	30 – 35	32.5	22	-2	-44	35 – 40	37.5	16	-1	-16	40 – 45	42.5	6	0	0	45 – 50	47.5	5	1	5	50 – 55	52.5	3	2	6	55 – 60	57.5	4	3	12			70		-79	For correct table 2 Marks
C.I.	x	f	$u = \frac{x - 42.5}{5}$	fu																																											
25 – 30	27.5	14	-3	-42																																											
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40 – 45	42.5	6	0	0																																											
45 – 50	47.5	5	1	5																																											
50 – 55	52.5	3	2	6																																											
55 – 60	57.5	4	3	12																																											
		70		-79																																											

	$\text{Mean} = 42.5 - \frac{79}{70} \times 5 = 36.86$	1
	<p>SECTION D</p> <p>This section comprises of long answer (LA) type questions of 5 marks each.</p>	
32.	<p>One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find :</p> <p>(a) The height of the basket from the ground. (b) The distance of the basket from the first observer's eye. (c) The horizontal distance of the second observer from the basket.</p>	
Sol.	 <p>Let B is the basket of hot air balloon</p> $\tan 60^\circ = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \quad \text{(i)}$ $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{x + 100} \Rightarrow x = h\sqrt{3} - 100 \quad \text{(ii)}$	<p>Correct Figure 1 Mark</p>

using (i) and (ii)

(a) $h = (h\sqrt{3} - 100)\sqrt{3} = 3h - 100\sqrt{3} \Rightarrow h = 50\sqrt{3} \text{ m}$

$\frac{1}{2}$

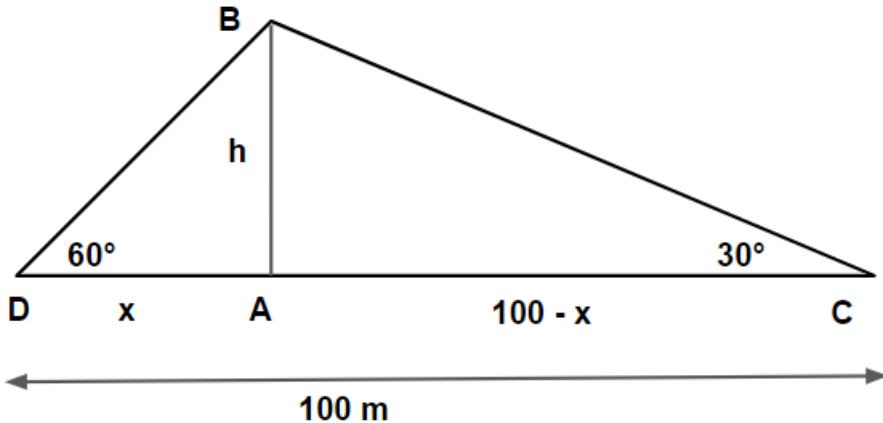
(b) $\sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{h}{y} = \frac{50\sqrt{3}}{y} \Rightarrow y = 100 \text{ m}$

1

(c) $x = \frac{h}{\sqrt{3}} = 50 \text{ m} \Rightarrow x + 100 = 150 \text{ m}$

$\frac{1}{2}$

ANOTHER SOLUTION AS PER BELOW FIGURE IS ALSO POSSIBLE



Correct Figure

1 Mark

Let B is the basket of hot air balloon. D and C be the positions of the first and second observer's respectively.

$$\tan 60^\circ = \sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \quad \text{(i)}$$

1

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{100-x} \Rightarrow \sqrt{3}h = 100 - x \quad \text{(ii)}$$

1

(a) using (i) and (ii)

$$h = \sqrt{3} (100 - \sqrt{3}h) \Rightarrow h = 25\sqrt{3} \text{ m}$$

$\frac{1}{2}$

$$(b) \sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{h}{BD} \Rightarrow BD = 50 \text{ m}$$

1

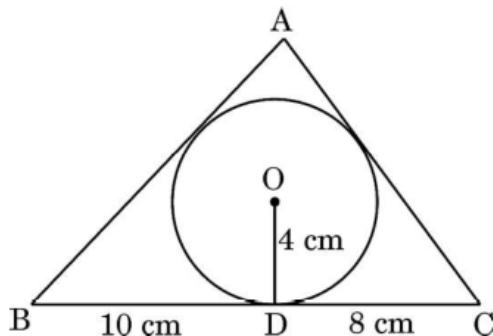
$$(c) x = \frac{h}{\sqrt{3}} = 25 \text{ m}$$

$\frac{1}{2}$

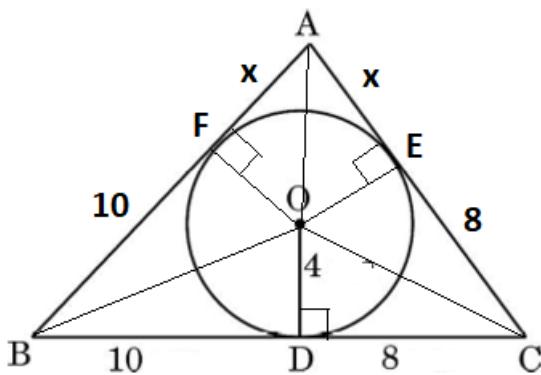
$$\Rightarrow AC = 100 - x = 75 \text{ m}$$

33(a).

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides AB and AC, if it is given that area $\Delta ABC = 90 \text{ cm}^2$.



Sol.



	<p>Join OA, OB, OC and draw OE \perp AC and OF \perp AB.</p> <p>$BF = 10 \text{ cm}$, $CE = 8 \text{ cm}$, Let $AF = AE = x$</p> <p>$\text{ar } \triangle ABC = \text{ar } \triangle BOC + \text{ar } \triangle COA + \text{ar } \triangle AOB$</p> <p>$90 = \frac{1}{2} \cdot 4 (BC + CA + AB)$</p> <p>$90 = 2(18 + 8 + x + 10 + x)$</p> <p>$90 = 4(18 + x)$</p> <p>$x = 4.5$</p> <p>$AB = 14.5 \text{ cm}$ and $AC = 12.5 \text{ cm}$</p>	$1\frac{1}{2}$ $1\frac{1}{2}$ 1 1
	OR	
33(b).	<p>Two circles with centres O and O' of radii 6 cm and 8 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.</p>	

Sol.	$OO' = \sqrt{6^2 + 8^2} = 10 \text{ cm}$ $\{OP \perp O'P\}$ <p>Let $OA = x$, $O'A = 10 - x$</p> $AP^2 = 36 - x^2$ <p>Also $AP^2 = 64 - (10 - x)^2$</p> <p>Therefore $36 - x^2 = 64 - (10 - x)^2$</p> $\Rightarrow 36 - x^2 = 64 - 100 + 20x$ $\Rightarrow x = 3.6$ <p>In ΔPAO, $AP^2 = 36 - (3.6)^2 = 23.04$</p> $\Rightarrow AP = 4.8$ <p>Length $PQ = 2 \times AP = 9.6 \text{ cm}$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 2 1 $\frac{1}{2}$
34(a).	A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed ?	
Sol.	<p>Let first average speed of the train be $x \text{ km/hr.}$</p> $\frac{54}{x} + \frac{63}{x+6} = 3$ $\Rightarrow 54x + 324 + 63x = 3x^2 + 18x$	2

	$\Rightarrow 3x^2 - 99x - 324 = 0 \text{ or } x^2 - 33x - 108 = 0$ $\Rightarrow (x - 36)(x + 3) = 0$ $\Rightarrow x = 36, -3 \text{ (rejected)}$	2
	<p>Therefore, first average speed of the train was 36 km/hr.</p>	1
	<p>OR</p>	
34(b).	<p>Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.</p>	
Sol.	<p>Let the time taken by smaller diameter tap be x hrs.</p>	
	<p>Time taken by larger diameter tap is $(x - 2)$ hrs.</p>	
	<p>Therefore $\frac{1}{x-2} + \frac{1}{x} = \frac{8}{15}$</p>	2
	$\Rightarrow 15(2x - 2) = 8x(x - 2)$	
	$\Rightarrow 8x^2 - 46x + 30 = 0$	
	$\Rightarrow 4x^2 - 23x + 15 = 0$	1
	$\Rightarrow (4x - 3)(x - 5) = 0$	
	$\Rightarrow x = \frac{3}{4}, x = 5$	1

	$x \neq \frac{3}{4}$ as $x - 2 < 0$ Smaller diameter tap fills in 5 hrs. Larger diameter tap fills in 3 hrs.	1
35.	A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use $\pi = 3.14$)	
Sol.	Area of that part of the field in which the horse can graze by means of a 5 m long rope $= \frac{1}{4} \times 3.14 \times (5)^2$ $= 19.625 \text{ m}^2$ Area of that part of the field in which the horse can graze by means of a 10 m long rope $= \frac{1}{4} \times 3.14 \times (10)^2$ $= 78.5 \text{ m}^2$ Increase in grazing area $= 78.5 \text{ m}^2 - 19.625 \text{ m}^2 = 58.875 \text{ m}^2$	1 1 1 1 1
	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	

36.

A golf ball is spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.



Based on the above, answer the following questions :

- (i) Find the surface area of one such dimple.
- (ii) Find the volume of the material dug out to make one dimple.
- (iii) (a) Find the total surface area exposed to the surroundings.

OR

- (iii) (b) Find the volume of the golf ball.

Sol.

$$(i) SA = 2\pi r^2 = 2 \times \frac{22}{7} \times 4 = \frac{176}{7} \text{ mm}^2 \text{ or } 25.1 \text{ mm}^2$$

1

$$(ii) \text{ Volume of material dug out to make one dimple} = \frac{2}{3} \times \frac{22}{7} \times 8$$

$$= \frac{352}{21} \text{ mm}^3 \text{ or } 16.76 \text{ mm}^3$$

1

(iii)(a) radius of ball = 21 mm

Total surface area exposed to surroundings

$$= 4\pi(21)^2 - 315 \times \pi(2)^2 + 315 \times 2\pi(2)^2$$

1

$$= 4 \times \frac{22}{7} \times 21 \times 21 + \frac{22}{7} \times 315 \times 4$$

1

$$= 9504 \text{ mm}^2$$

OR

(iii) (b) Volume of the golf ball = $\frac{4}{3}\pi(21)^3 - 315 \times \frac{2}{3}\pi(2)^3$

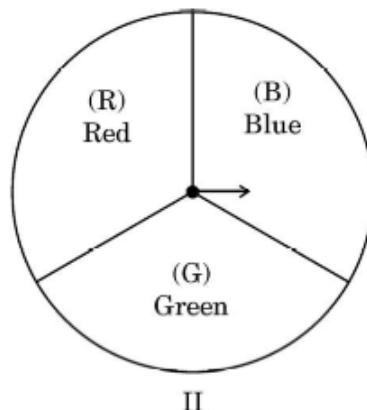
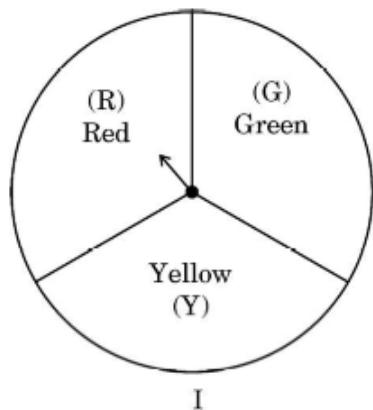
1

$$= 33528 \text{ mm}^3$$

1

37.

A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.



Making Purple : Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.

Based on the above, answer the following questions :

- List all possible outcomes of the game.
- Find the probability of 'Making Purple'.
- (a) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school.

If 99 participants played, calculate how much fund could the school have collected.

OR

- (b) If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school ? (Number of participants = 99)

Sol. (i) All possible outcomes: RR, RG, RB, GR, GB, GG, YR, YB, YG

1

(ii) Number of favourable outcome (RB) = 1

$P(\text{Making purple}) = \frac{1}{9}$	1
(iii)(a) As $P(\text{winning}) = \frac{1}{9}$	$\frac{1}{2}$
Therefore, number of people must win = $\frac{1}{9} \times 99 = 11$	$\frac{1}{2}$
\therefore Game lost by 88 persons.	$\frac{1}{2}$
Funds collected = $5 \times 88 - 10 \times 11 = ₹ 330$	1
OR	
(iii)(b) Number of participants = 99	
$P(\text{winning the game}) = \frac{1}{9}$	
Number of persons won = 11	$\frac{1}{2}$
Number of persons lost = 88	$\frac{1}{2}$
Funds collected = $88 \times 5 - 11 \times 5 = ₹ 385$	1

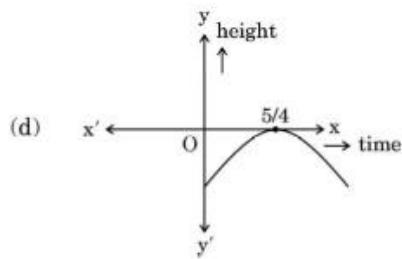
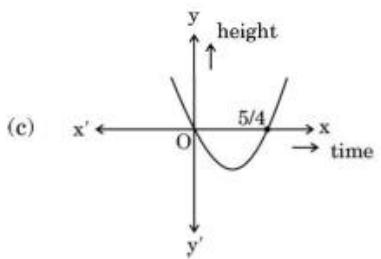
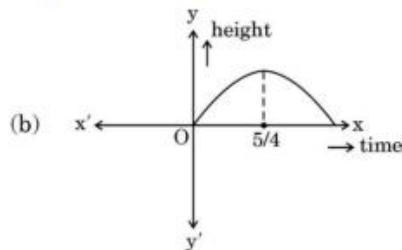
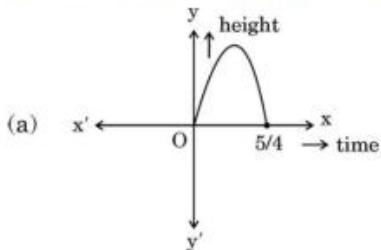
38.

In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.



Based on the above, answer the following questions :

(i) Find zeroes of polynomial $p(t) = 20t - 16t^2$.
(ii) Which of the following types of graph represents $p(t)$?



(iii) (a) What would be the value of h at $t = \frac{3}{2}$? Interpret the result.

OR

(iii) (b) How much distance has the dolphin covered before hitting the water level again ?

Sol.	<p>(i) $-16t^2 + 20t = 0 \Rightarrow 4t(-4t + 5) = 0$</p> <p>$t = 0, t = \frac{5}{4}$</p> <p>(ii) (a)</p> <p>(iii)(a) At $t = \frac{3}{2}$, $h = -16 \times \frac{9}{4} + 20 \times \frac{3}{2} = -36 + 30 = -6$</p> <p>It means after $\frac{3}{2}$ seconds, dolphin has reached 6 cm below water level.</p> <p>OR</p> <p>(iii)(b) Speed of dolphin = 20 cm per second.</p> <p>In one second, distance covered = 20 cm</p> <p>In $\frac{5}{4}$ seconds, distance covered = $20 \times \frac{5}{4} = 25$ cm</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p>
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