Class X Session 2025-26
Subject - Mathematics (Standard)
Sample Question Paper - 10

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are AssertionReason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E .
9. Draw neat and clean figures wherever required.
10. Take wherever required if not stated.
11. Use of calculators is not allowed.

Section A

1. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number is: **[1]**

a) 36 b) 45

c) 81 d) 9

2. If $n = 2^3 \times 3^4 \times 5^4 \times 7$, then the number of consecutive zeros in n, where n is a natural number, is **[1]**

a) 7 b) 3

c) 2 d) 4

3. If α and β are the zeroes of the polynomial $\mathrm{p(x)} = \mathrm{kx}^2 - 30\mathrm{x} + 45\mathrm{k}$ and $\alpha + \beta = \alpha\beta$, then the value of k is: **[1]**

a) $\frac{3}{2}$ b) $-\frac{3}{2}$

c) $-\frac{2}{3}$ d) $\frac{2}{3}$

4. If one zero of the quadratic polynomial x^2 - 5x + k is -4 , then the value of k is **[1]**

a) 18 b) -18

c) -36 d) 36

5. The value of **k** for which the system of equations kx + 2y = 5 and 3x + 4y = 1 have no solution, is **[1]**

a) k = 15
b) $k \neq \frac{2}{3}$
c) $k \neq \frac{3}{2}$
d) $k = \frac{3}{2}$

6. The area of the triangle formed by the lines x = 3, y = 4 and x = y is **[1]**

a) 3sq. unit
b) 1/2 sq. unit
c) 2sq. unit
d) 1 sq. unit

7. $2x^2 - 3x + 2 = 0$ have **[1]**

a) Real and Distinct roots
b) Real roots
c) No Real roots
d) Real and Equal roots

8. The value(s) of k for which the quadratic equation $5x^2 - 9kx + 5 = 0$ has real and equal roots, is/are: **[1]**

a) $\pm\frac{10}{9}$
b) $\frac{-10}{9}$
c) $\pm\frac{9}{10}$
d) $\frac{10}{9}$

9. If in an A.P., a = 2 and $S_{10} = 335$, then its 10^{th} term is: **[1]**

a) 58
b) 55
c) 65
d) 68

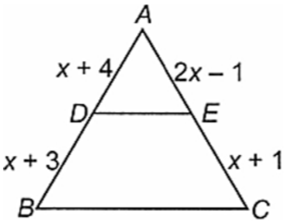
10. If a, b, c, l, m are in A.P., then the value of a - 4b + 6c - 4l + m is **[1]**

a) 0
b) 3
c) 1
d) 2

11. $\triangle PQR \sim \triangle XYZ$ and the perimeters of and $\triangle PQR$, $\triangle XYZ$ are 30 cm and 18 cm respectively. If QR = 9 cm, then, YZ is equal to **[1]**

a) 4.5 cm
b) 12.5 cm
c) 5.4 cm
d) 9.5 cm

12. In the given figure, DE || BC. Find the value of X. **[1]**



a) $\sqrt{3}$
b) $\sqrt{5}$
c) $\sqrt{7}$
d) $\sqrt{6}$

13. If the points (6, 1), (8, 2), (9, 4) and (p, 3), taken in order are the vertices of a parallelogram, then the value of ‘p’ is **[1]**

a) 6
b) – 7
c) 7
d) 5

14. The distance between the points (a, a) and $(-\sqrt{3}a, \sqrt{3}a)$ is **[1]**

a) $2\sqrt{2}$ units

b) $2a\sqrt{2}$ units

c) $3\sqrt{2}a$ units

d) 2 units

15. If $\sqrt{3}\tan\theta = 3\sin\theta$, ($\theta \neq 0$) then the value of $\sin^2\theta - \cos^2\theta$ is **[1]**

a) 0

b) 1

c) $\frac{1}{2}$

d) $\frac{1}{3}$

16. $\sec^4$A - $\sec^2$A is equal to **[1]**

a) $\tan^2$ A - $\tan^4$A

b) $\tan^2$A + $\tan^3$A

c) $\tan^4$A + $\tan^2$A

d) $\tan^4$A - $\tan^2$A

17. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is **[1]**

a) 7

b) 10

c) 12

d) 5

18. A cubical block of side 7 cm is surmounted by a hemisphere. The greatest diameter of the hemisphere is **[1]**

a) 3.5cm

b) 10.5cm

c) 7cm

d) 14cm

19. **Assertion (A):** The arithmetic mean of the following given frequency distribution table is 13.81. **[1]**

x	4	7	10	13	16	19
f	7	10	15	20	25	30

Reason (R): $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** In a cricket match, a batsman hits a boundary 9 times out of 45 balls he plays. The probability that in a given ball, he does not hit the boundary is $\frac{4}{5}$. **[1]**

Reason (R): P(E) + P(not E) = 1

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Prove that 11 + $3\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. **[2]**

22. Find the values of **a** and **b** for which the system of linear equations 3x + 4y = 12, (a + b) x + 2 (a − b) y = 24 has infinite number of solutions. **[2]**

23. Name the type of quadrilateral formed, if any, by the points (4, 5), (7, 6), (4, 3), (1, 2), and give a reason for your answer. **[2]**

OR

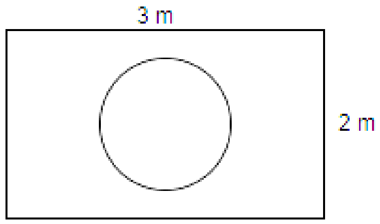
Find the coordinates of the point which divides the line segment joining the points (7, -1) and (-3,- 4) internally in the ratio 2 : 3.

24. Prove that: $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$ **[2]**

OR

Prove that: $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$

25. Suppose you drop a die at random on the rectangular region shown in Figure. What is the probability that it will land inside the circle with diameter 1m? **[2]**



Section C

26. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 +bx + c, then evaluate: $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$. **[3]**

OR

If α and β are the zeros of the polynomial f(x) = x^2 - 5x + k such that $\alpha - \beta$ = 1, find the value of k.

27. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y. **[3]**

28. Prove the following identity : $\frac{1}{\cot^2\theta} + \frac{1}{1+\tan^2\theta} = \frac{1}{1-\sin^2\theta} - \frac{1}{cosec^2\theta}$ **[3]**

29. Find the missing frequencies in the following frequency distribution table, if N = 100 and median is 32. **[3]**

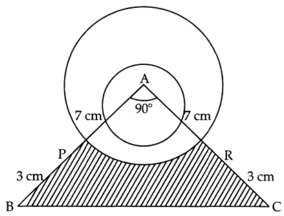
Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	Total
Number of students	10	?	25	30	?	10	100

OR

Find the mean marks per student, using assumed-mean method:

Marks	0 -10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Number of students	12	18	27	20	17	6

30. A momento is made as shown in the figure. Its base PBCR is silver plated from the front side. Find the area which is silver plated. $(\pi = \frac{22}{7})$ **[3]**



31. ABC is a right-angled triangle, right angled at A. A circle is inscribed in it. The lengths of two sides containing the right angle are 24 cm and 10 cm. Find the radius of the incircle. **[3]**

Section D

32. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day. Represent the situations mathematically (quadratic equation). **[5]**

OR

A train travels a distance of 90 km at a constant speed. Had the speed been 15 km/h more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

33. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio. **[5]**

34. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the further time taken by the car to reach the foot of the tower from this point. **[5]**

OR

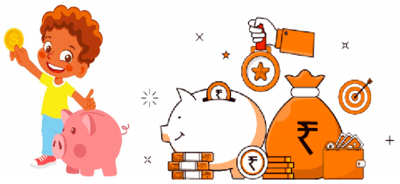
An aeroplane when flying at a height of 3000 metres from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. [Take $\sqrt{3}$=1.73]

35. An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar if one cubic cm of iron weighs 7.8 grams. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



i. If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it. (1)
ii. Find the total money he saved. (1)
iii. How much money Akshar saves in 10 days? (2)

OR

How many coins are there in piggy bank on 15th day? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Under the physical and health education a medical check up program was conducted in a Vidyalaya to improve the health and fitness conditions of the students. Reading of the heights of 50 students was obtained as given in the table below:

Height (in cm)	**Number of students**
135-140	2

140-145	8
145-150	10
150-155	15
155-160	6
160-165	5
165-170	4

i. Find the lower class limit of the modal class. (1)

ii. Find the median class. (1)

iii. Find the assumed mean of the data. (2)

OR

Find the median of the given data. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Essel World is one of India's largest amusement parks that offers a diverse range of thrilling rides, water attractions and entertainment options for visitors of all ages. The park is known for its iconic ''Water Kingdom'' section, making it a popular destination for family outings and fun-filled adventure. The ticket charges for the park are ₹ 150 per child and ₹ 250 per adult.

On a day, the cashier of the park found that 300 tickets were sold and an amount of ₹ 55,000 was collected.

Based on the above, answer the following questions:

i. If the number of children visited be x and the number of adults visited be y, then write the given situation algebraically. **(1)**

ii. a. How many children visited the amusement park that day? **(2)**

OR

b. How many adults visited the amusement park that day? **(2)**

iii. How much amount will be collected if 250 children and 100 adults visit the amusement park? **(1)**

Solution

Section A

1.

(c) 81

Explanation:

Let the two numbers be x and y.

It is given that:

x = 54

HCF = 27

LCM = 162

We know,

x $\times$ y = HCF $\times$ LCM

$\Rightarrow$ 54 $\times$ y = 27 $\times$ 162

$\Rightarrow$ 54y = 4374

$\Rightarrow$ $\therefore y = \frac{4374}{54}$ = 81

2.

(b) 3

Explanation:

Since, it is given that

$n = 2^3 \times 3^4 \times 5^4 \times 7$

$= 2^3 \times 5^4 \times 3^4 \times 7$

$= 2^3 \times 5^3 \times 5 \times 3^4 \times 7$

$= (2 \times 5)^3 \times 5 \times 3^4 \times 7$

$= 5 \times 3^4 \times 7 \times (10)^3$

So, this means the given number n will end with 3 consecutive zeroes.

3.

(d) $\frac{2}{3}$

Explanation:

$\alpha + \beta = \frac{-b}{a}$

$= \frac{-(-30)}{k}$

$\alpha\beta = \frac{c}{a} = \frac{45k}{k}$

ATQ

$\frac{30}{K} = \frac{45}{K}K$

$K = \frac{30}{45}$

$K = \frac{2}{3}$

4.

(c) -36

Explanation:

p(-4) = 0(since - 4 is root of p(x))

$(-4)^2$ - 5(-4) + k = 0

$\Rightarrow$ 16 + 20 + k = 0

36 + k = 0

k = -36

5.

(d) k = $\frac{3}{2}$

Explanation:

kx + 2y - 5 = 0

3x + 4y - 1 = 0

For No Solution

$\frac{k}{3}$ = $\frac{2}{4}$ $\neq$ $\frac{-5}{-1}$

k = $\frac{6}{4}$ = $\frac{3}{2}$

k = $\frac{3}{2}$

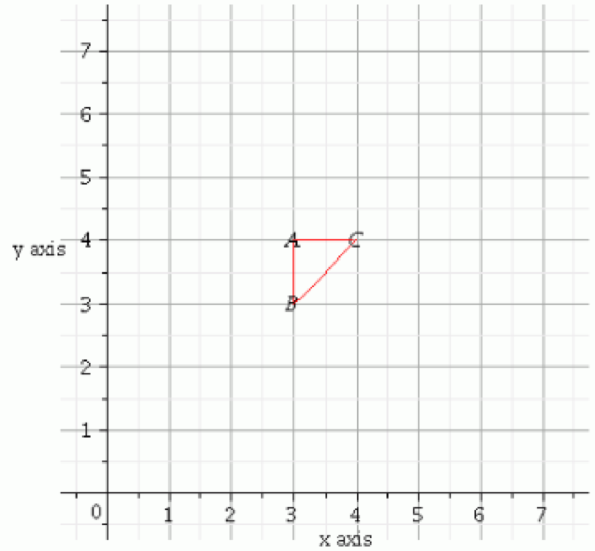
6.

(b) 1/2 sq. unit

Explanation:

Given x = 3, y = 4 and x = y

We have plotting points as (3,4), (3,3), (4,4) when x = y



Therefore, area of $\triangle$ABC = $\frac{1}{2}$(Base $\times$ Height) = $\frac{1}{2}$ (AB $\times$ AC) = $\frac{1}{2}$ (1 $\times$ 1) = $\frac{1}{2}$

Area of triangle ABC is $\frac{1}{2}$ square units.

7.

(c) No Real roots

Explanation:

D = b^2 - 4ac

D = $(-3)^2$ - 4 $\times$ 2 $\times$ 2

D = 9 - 16

D = - 7

D < 0. Hence No Real roots.

8. **(a)** $\pm\frac{10}{9}$

Explanation:

$5x^2 - 9kx + 5 = 0$ has real and equal roots, if

$b^2 - 4ac = 0$

$\Rightarrow (-9k)^2 - 4(5)(5) = 0$

$\Rightarrow 81k^2 - 100 = 0$

$\Rightarrow k^2 = \frac{100}{81}$

$\Rightarrow k = \pm\frac{10}{9}$

9.

(c) 65

Explanation:

a = 2, S_{10} = 335, a_{10} = ?

$S_{10} = \frac{10}{2}[2 \times 2 + (9)(d)]$

335 = 5[4 + 9d]

67 = 4 + 9d

63 = 9d

d = 7

a_{10} = a + 9d

= 2 + 9 $\times$ 7

= 2 + 63

a_{10} = 65

10. **(a)** 0

Explanation:

Given: a, b, c, l, m are in A.P.

Therefore,

a + m = 2c ... (i)

b + l = 2c ... (ii)

a - 4b + 6c - 4l + m

= a + m + 6c - 4b - 4l

= a + m + 6c - 4(b + l)

substituting from (i) and (ii)

= 2c + 6c - 8c

= 0

11.

(c) 5.4 cm

Explanation:

let the sides of triangle PQR be a,b,9 and triangle XYZ be ax, bx, 9x (triangles are similar, hence their sides are in proportion

a + b + 9 = 30, therefore a + b = 21, ax + bx + 9x = 18,

x = $\frac{18}{30}$

$\Rightarrow$ YZ = 9x = 5.4

12.

(c) $\sqrt{7}$

Explanation:

In $\triangle$ABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x+4}{x+3} = \frac{2x-1}{x+1}$

$\Rightarrow$ (2x - 1) (x + 3) = (x + 4) (x + 1)

$\Rightarrow 2x^2 + 6x - x - 3 = x^2 + x + 4x + 4$

$\Rightarrow 2x^2 + 5x - 3 = x^2 + 5x + 4$

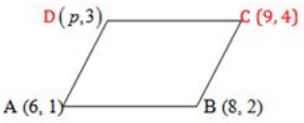
$\Rightarrow x^2 = 7 \Rightarrow x = \sqrt{7}$ ($\because x \neq -\sqrt{7}$)

13.

(c) 7

Explanation:

In parallelogram, AB = CD, squaring both sides



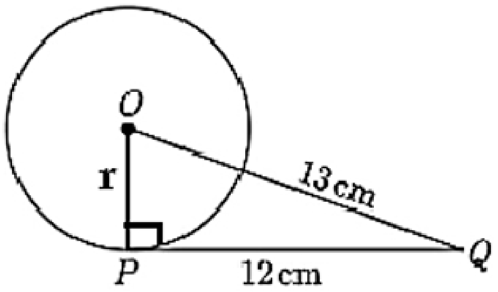
$\Rightarrow$ $AB^2 = CD^2$
$\Rightarrow (8-6)^2 + (2-1)^2 = (p-9)^2 + (3-4)^2$
$\Rightarrow 4 + 1 = p^2 + 81 - 18p + 1$
$\Rightarrow p^2 - 18p + 77 = 0$
$\Rightarrow (p-7)(p-11) = 0$
$\Rightarrow p = 7$ and $p = 11$

14.
(b) $2a\sqrt{2}$ units
Explanation:
Let the points be A(a, a) and B$(-\sqrt{3}a, \sqrt{3}a)$
$\therefore$AB = $\sqrt{\left(-\sqrt{3}a - a\right)^2 + \left(\sqrt{3}a - a\right)^2}$
= $\sqrt{3a^2 + a^2 + 2\sqrt{3}aa + 3a^2 + a^2 - 2\sqrt{3}aa}$
= $\sqrt{6a^2 + 2a^2}$
= $\sqrt{8a^2}$
= $2a\sqrt{2}$ units

15.
(d) $\frac{1}{3}$
Explanation:
Given: $\sqrt{3}\tan\theta = 3\sin\theta$
$\Rightarrow \sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$
$\Rightarrow \frac{\sqrt{3}}{3} = \cos\theta$
$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$
And $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$
$\therefore \sin^2\theta - \cos^2\theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

16.
(c) $\tan^4A + \tan^2A$
Explanation:
We have, $\sec^4A - \sec^2A = \sec^2A(\sec^2A - 1)$
$= (1 + \tan^2A)\tan^2A$
$= \tan^2A + \tan^4A$
$= \tan^4A + \tan^2A$

17.
(d) 5
Explanation:



∵ We know that

PQ ⊥ OP

∴ △QPO is right angled △

∴ By python theory.

$QO^2 = QP^2 + OP^2$

$(13)^2 = (12)^2 + OP^2$

$r^2 = 169 - 144.$

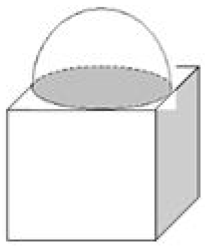
$r^2 = 25$

r = 5 cm

18.

(c) 7cm

Explanation:



It is clear that Maximum diameter of hemisphere can be the side of the cube.

∴ The greatest diameter of the hemisphere = 7 cm

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Favourable case he does not hit boundary = 45 - 9 = 36

p(he does not hit boundary) $= \frac{36}{45} = \frac{4}{5}$

p(he hit boundary) $= \frac{9}{45} = \frac{1}{5}$

from above we can say that

p(E) + p(not E) $= \frac{4}{5} + \frac{1}{5} = \frac{5}{5} = 1$

Section B

21. Let us assume that $11 + 3\sqrt{2}$ be a rational number.

$\Rightarrow 11 + 3\sqrt{2} = \frac{a}{b}$, where a and b are integers, b $\neq$ 0

$\Rightarrow \sqrt{2} = \frac{a-11b}{3b}$

RHS is a rational number but LHS is irrational.

∴ Our assumption was wrong. Hence, $11 + 3\sqrt{2}$ is an irrational number.

22. For Infinite number of solutions $\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{12}{24}$

$\frac{3}{a+b} = \frac{1}{2} \Rightarrow$ a + b = 6

$\frac{2}{a-b} = \frac{1}{2} \Rightarrow$ a - b = 4

On solving, a = 5, b = 1

23. (4, 5), (7, 6), (4, 3), (1, 2)

Let A $\rightarrow$ (4, 5), B $\rightarrow$ (7, 6), C $\rightarrow$ (4, 3) and D $\rightarrow$ (1, 2)

Then, $AB = \sqrt{(7-4)^2 + (6-5)^2}$

$= \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2 + (3-6)^2}$

$\sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(1-4)^2 + (2-3)^2}$

$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

$DA = \sqrt{(4-1)^2 + (5-2)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(4-4)^2 + (3-5)^2} = 2$

$BD = \sqrt{(1-7)^2 + (2-6)^2}$

$= \sqrt{36+16} = \sqrt{52}$

We see that

AB = CD, opposite sides are equal

BC = DA

and AC $\neq$ BD ..... Diagonals are unequal

Hence, the quadrilateral ABCD is a parallelogram.

OR

Given (7, -1) and (-3, -4)

So $x_1 = 7$, $y_1 = -1$

$x_2 = -3$, $y_2 = -4$

Using section formula

$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

$x = \frac{2(-3)+3(7)}{2+3} = \frac{-6+21}{5}$

$x = \frac{15}{5}$ = 3

Also $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

$y = \frac{2(-4)+3(-1)}{2+3}$

$y = \frac{-8-3}{5} = \frac{-11}{5}$

So coordinates of intersection point (3, $\frac{-11}{5}$)

24. LHS $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

Dividing numerator and denominator by cosθ

$= \frac{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sec\theta+1+\tan\theta}{\sec\theta+1-\tan\theta}$

Multiplying and dividing by $\sec\theta + 1 + \tan\theta$

$= \frac{\sec\theta+1+\tan\theta}{\sec\theta+1-\tan\theta} \times \frac{\sec\theta+1+\tan\theta}{\sec\theta+1+\tan\theta}$

$= \frac{(\sec\theta+1+\tan\theta)^2}{(\sec\theta+1)^2-\tan^2\theta}$

$= \frac{\sec^2\theta+1+\tan^2\theta+2\sec\theta+2\tan\theta+2\sec\theta\tan\theta}{1+\sec^2\theta+2\sec\theta-\tan^2\theta}$

Now, $1+\tan^2\theta = \sec^2\theta$

$= \frac{2\sec^2\theta+2\sec\theta+2\tan\theta+2\sec\theta\tan\theta}{2+2\sec\theta}$

$= \frac{2[\sec\theta(\sec\theta+1)+\tan\theta(1+\sec\theta]}{2(1+\sec\theta)}$

$= \frac{(\sec+\tan\theta)(\sec\theta+1)}{(1+\sec\theta)}$

$= \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$
$= \frac{1+\sin\theta}{\cos\theta} = RHS$

OR

$\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

R.H.S. = $\tan^2\theta.\sin^2\theta$

= $\tan^2\theta(1 - \cos^2\theta)$ $\left[\because \sin^2\theta = 1 - \cos^2\theta\right]$

= $\tan^2\theta - \tan^2\theta\cos^2\theta$

$= \tan^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta \left[\because \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right]$

= $\tan^2\theta - \sin^2\theta$= L.H.S.

Hence proved.

25. Total area of the given figure (rectangle) = $3 \times 2 = 6m^2$

d =1

$r = \frac{1}{2}$

And Area of circle = {tex}\mathrm{πr}^2{/tex}= $\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}\mathrm{m}^2$

$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

Hence, P(die to land inside the circle) = $\frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$

Hence the probability of die to land inside the circle is $\frac{\pi}{24}$

Section C

26. Since,

Sum of the zeroes of polynomial = α + β $= \frac{-b}{a}$

and product of zeroes of polynomial = αβ $= \frac{c}{a}$

Simplify the given expression and substitutie the values, we obtain

$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$

$= \frac{\alpha\beta^2+b\beta+\alpha^2a+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \frac{a\alpha^2+b\beta^2+b\alpha+b\beta}{a^2\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$

$= \frac{a\left[(\alpha+\beta)^2+b(\alpha+\beta)\right]}{ac}$

$= \frac{a\left[(a+\beta)^2-2\alpha\beta\right]-\frac{b^2}{a}}{ac}$

$= \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac}$

$= \frac{a\left[\frac{b^2-ac}{a}\right]-\frac{b^2}{a}}{ac}$

$= \frac{a\left[\frac{b^2-ac-b^2}{a}\right]}{ac}$

$= \frac{b^2-2c-b^2}{ac}$

$= \frac{-2c}{ac} = \frac{-2}{a}$

OR

Since α, β are the zeros of the polynomial f(x) = x^2 - 5x + k.

Compare f(x) = x^2 - 5x + k with ax^2 + bx + c.

So, a = 1 , b = -5 and c = k

$\alpha + \beta = -\frac{(-5)}{1}$ = 5

$\alpha\beta = \frac{k}{1} = k$

Given, $\alpha - \beta$ = 1

Now, $(\alpha+\beta)^2 = (\alpha-\beta)^2 + 4\alpha\beta$

⇒ $(5)^2 = (1)^2 + 4k$

⇒ 25 = 1 + 4k

⇒ 4k = 24

$\Rightarrow$ k = 6

Hence the value of k is 6.

27. Let A → (1, 2), B → (4, y), C→ (x, 6) and D→ (3, 5).

We know that the diagonals of parallelogram bisect each other.

So, Coordinates of the mid-point of diagonal AC

= Coordinates of the mid-point of diagonal BD

$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$

$\Rightarrow \left(\frac{1+x}{2}, 4\right) = \left(\frac{7}{2}, \frac{y+5}{2}\right)$

$\Rightarrow \frac{1+x}{2} = \frac{7}{2}$

$\Rightarrow$ 1 + x = 7

$\Rightarrow$ x = 6

and $4 = \frac{y+5}{2}$

$\Rightarrow$ y + 5 = 8

$\Rightarrow$ y = 3

28. LHS

$= \frac{1}{\cot^2\theta} + \frac{1}{1+\tan^2\theta}$

$= \tan^2\theta + \frac{1}{\sec^2\theta}$

$= \tan^2\theta + \cos^2\theta$

$= (\sec^2\theta - 1) + \cos^2\theta$

$= \sec^2\theta - (1 - \cos^2\theta)$

$= \sec^2\theta - \sin^2\theta$

$= \frac{1}{\cos^2\theta} - \sin^2\theta$

$= \frac{1}{1-\sin^2\theta} - \frac{1}{cosec^2\theta}$

= R.H.S

Hence proved.

29. Let f_1 and f_2 be the missing frequencies.

$10 + f_1 + 25 + 30 + f_2 + 10 = 100$

$\Rightarrow f_1 + f_2 = 25$

Median is 32, which lies in 30-40. So, the median class is 30-40.

$\therefore l = 30, h = 10, f = 30, N = 100$ and cf = $10 + f_1 + 25 = f_1 + 35$.

Now, Median, M $= l + \left\{h \times \frac{\left(\frac{N}{2} - cf\right)}{f}\right\}$

$\Rightarrow \quad 30 + \left[10 \times \frac{50-(f_1+35)\}}{30}\right] = 32$

$\Rightarrow \quad 30 + \frac{(15-f_1)}{3} = 32$

$\Rightarrow (15 - f_1) = 6$

$\Rightarrow f_1 = 9$

$f_2 = 25 - 9 = 16$

Hence, $f_1 = 9$ and $f_2 = 16$.

OR

The assumed mean is 25.

Class Interval	**Frequency(f_i)**	**Mid value x_i**	**Deviation** $d_i = (x_i - 25)$	$(f_i \times d_i)$
0 - 10	12	5	-20	-240
10 - 20	18	15	-10	-180
20 - 30	27	25 = A	0	0
30 - 40	20	35	10	200

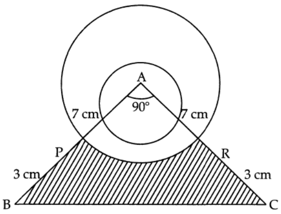
40 - 50	17	45	20	340
50 - 60	6	55	30	180
	$\Sigma f_i = 100$			$\Sigma\left(f_i \times d_i\right) = 300$

we know that, mean= $A + \frac{\sum(f_i \times x_i)}{\sum f_i}$

From table, $\Sigma f_i = 100$ and $\Sigma\left(f_i \times d_i\right)$= 300

Therefore, mean $\bar{x} = \left(25 + \frac{300}{100}\right)$

$= 25 + 3 = 28$

30. 

Base = 7 + 3 = 10cm and height = 7 + 3 =10 cm

From the given figure

Area of right-angled$\triangle ABC = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 10 \times 10$

$= 50 \text{ cm}^2$

Area of quadrant APR of the circle of radius 7 cm

$= \frac{1}{4} \times \pi \times (7)^2$

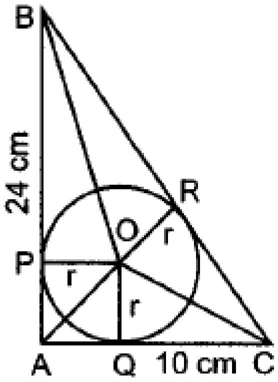
Area of quadrant= $\frac{1}{4} \times \frac{22}{7} \times 49 = 38.5\text{cm}^2$

Area of base PBCR = Area of $\triangle$ABC - Area of quadrant APR

= $50 - 38.5$ = 11.5 cm^2.

So, Area of shaded portion is 11.5 cm^2.

31. Given,



$AB = 24cm, AC = 10cm$

In right-angled $\triangle$ABC

$\text{BC}^2 = \text{AB}^2 + \text{AC}^2$

$= 24^2 + 10^2$

$= 676$

$\Rightarrow \mathbf{BC} = 26\text{cm}$

Let r be the radius of the incircle

$\Rightarrow$ OP $\perp$ AB, OQ $\perp$ AC and OR $\perp$ BC

OP = OQ = OR [Incentre of a triangle is equidistant from its sides]

ar($\triangle$ABC) = ar($\triangle$AOB) + ar($\triangle$BOC) + ar($\triangle$AOC)

$\frac{1}{2}\text{AB} \times \text{AC} = \frac{1}{2}\text{AB} \times \text{OP} + \frac{1}{2}\text{AC} \times \text{OQ} + \frac{1}{2} \times \text{BC} \times \text{OR}$

$\frac{1}{2} \times 24 \times 10 = \frac{1}{2}[24 \times r + 10 \times r + 26 \times r]$

$\Rightarrow\ 120 = r[24 + 10 + 26]$

$\Rightarrow$ 120 = r[24+ 10 + 26]

$\Rightarrow$ 120 = 30r $\Rightarrow$ r = 4 cm

Section D

32. Let the number of toys produced be x.

$\therefore$ Cost of production of each toy = Rs $(55 - x)$

It is given that, total production of the toys = Rs 750

$\therefore$ x(55 – x) = 750

$\Rightarrow x^2 - 55x + 750 = 0$

Now to factorize this equation we have to find two numbers such that their product is 750 and sum is 55

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

Either x – 25 = 0 or $x - 30 = 0$

i.e., $x = 25$ or $x = 30$

Hence, the number of toys will be either 25 or 30.

OR

Let the original speed of the train be x km/hr.

We know that time taken to cover 'd' km with speed 's' km/h $= \frac{d}{s}$ $\therefore$time taken to cover 90 km $= \frac{90}{x}$ hours

&, Time taken to cover 90 km when the speed is increased by 15 km/hr $= \frac{90}{x+15}$ hours

According to the question ;

$\frac{90}{x} - \frac{90}{x+15} = \frac{30}{60}$ (time reduced by 30 minutes with increased speed)

$\Rightarrow \frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$

$\Rightarrow \frac{90x+1350-90x}{x^2+15x} = \frac{1}{2}$

$\Rightarrow \frac{1350}{x^2+15x} = \frac{1}{2}$

$\Rightarrow 2700 = x^2 + 15x$

$\Rightarrow x^2 + 15x - 2700 = 0$

$\Rightarrow x^2 + 60x - 45x - 2700 = 0$

$\Rightarrow x(x + 60) - 45(x + 60) = 0$

$\Rightarrow (x + 60)(x - 45) = 0$

$\Rightarrow x + 60 = 0$ or $x - 45 = 0$

$\Rightarrow x = -60$ or $x = 45$

Since the speed cannot be negative, $x \neq -60$.

$\Rightarrow x = 45$

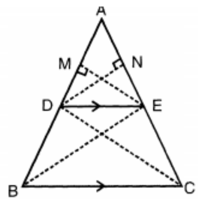
Thus, the original speed of the train is 45 km/hr.

33. Given: ABC is a triangle in which DE $\parallel$ BC.

To prove: $\frac{AD}{BD} = \frac{AE}{CE}$

Construction: Draw $DN \perp AE$ and $EM \perp AD$., Join BE and CD.

Proof :



In $\triangle ADE$,

Area of $\Delta ADE = \frac{1}{2} \times AE \times DN$ ...(i)

In ΔDEC,

Area of $\Delta DCE = \frac{1}{2} \times CE \times DN$ ...(ii)

Dividing equation (l) by equation (ii),

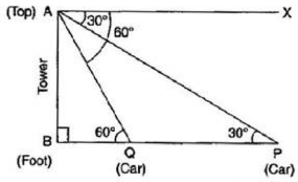
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$

$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEC)} = \frac{AE}{CE}$ ...(iii)

Similarly, In ΔADE,

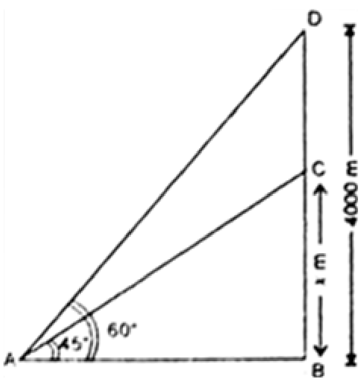
Area of $\Delta ADE = \frac{1}{2} \times AD \times EM$ ...(iv)

In ΔDEB,
Area of $\Delta DEB = \frac{1}{2} \times EM \times BD$ ...(v)
Dividing equation (iv) by equation (v),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AD}{BD}$ ...(vi)
ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.
$\therefore$ Area (ΔDEB) = Area (ΔDEC)
From equation (iii),
$\Rightarrow \frac{\text{area }(\Delta ADE)}{\text{area }(\Delta DEB)} = \frac{AE}{CE}$. ......(vii)
From equation (vi) and equation (vii),
$\frac{AE}{CE} = \frac{AD}{BD}$
$\therefore$ If a line is drawn parallel to one side of a triangle to intersect the other two sides in two points, then the other two sides are divided in the same ratio.

34. 

In right triangle ABP,
$\tan 30^\circ = \frac{\text{AB}}{\text{BP}}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\text{AB}}{\text{BP}}$
BP = AB$\sqrt{3}$ .... (i)
In right triangle ABQ,
$tan\ 60^0 = \frac{AB}{BQ}$
$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$
$\Rightarrow BQ = \frac{AB}{\sqrt{3}}$ .... (ii)
$\because$ PQ = BP - BQ
$\therefore$ PQ = AB$\sqrt{3} - \frac{AB}{\sqrt{3}} = \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}}$ = 2BQ [From eq. (ii)]
$\Rightarrow$ BQ = $\frac{1}{2}$PQ
$\because$ Time taken by the car to travel a distance PQ = 6 seconds.
$\therefore$ Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}$PQ = $\frac{1}{2} \times 6$ = 3 seconds.
Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

OR



Let C and D is the position of two aeroplanes. The height of the aeroplane which is at point D is 3000 m and it passes another aeroplane vertically which is at point C. Let BC = x m. It is also given that the angles of elevation of two planes from the point A on the ground is 45° and 60° respectively.
In right triangle ABC, we have
$tan 45^\circ = \frac{BC}{AB}$
$\Rightarrow 1 = \frac{x}{AB}$
$\Rightarrow x = AB$ ...(i)

In right triangle ABD, we have

$tan 60^\circ = \frac{BD}{AB}$

$\Rightarrow \sqrt{3} = \frac{3000}{AB}$

$\Rightarrow AB = \frac{3000}{\sqrt{3}}$ ...(ii)

Comparing (i) and (ii), we get

$x = \frac{3000}{\sqrt{3}}$

Hence, vertical distance between the aeroplane = CD = BD - BC

$= 3000 - \frac{3000}{\sqrt{3}}$

$= 3000 - \frac{3000}{1.732}$

$= 3000 - 1732.1$

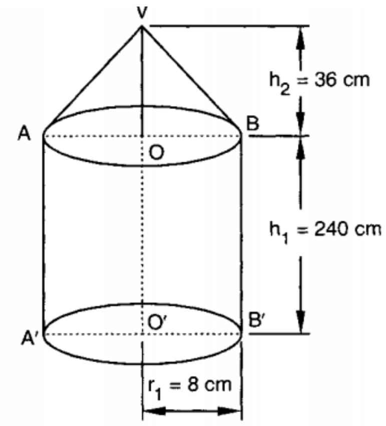
$= 1267.8$

Hence, the required distance between the two aeroplanes is =1267.8 m.

35. Let us suppose that r_1 cm and r_2 cm denote the radii of the base of the cylinder and cone respectively. Then,

$r_1 = r_2 = 8$ cm

Let us suppose that h_1 and h_2 cm be the heights of the cylinder and the cone respectively. Then,



$h_1 = 240$ cm and $h_2 = 36$ cm

$\therefore$ Volume of the cylinder = $\pi r_1^2 h_1 \text{cm}^3$

$= (\pi \times 8 \times 8 \times 240)\text{cm}^3$

$= (\pi \times 64 \times 240)\text{cm}^3$

Now, Volume of the cone = $\frac{1}{3}\pi r_2^2 h_2 \text{cm}^3$

$= \left(\frac{1}{3}\pi \times 8 \times 8 \times 36\right) \text{cm}^3$

$= \left(\frac{1}{3}\pi \times 64 \times 36\right) \text{cm}^3$

$\therefore$ Total volume of the iron = Volume of the cylinder + Volume of the cone

$= \left(\pi \times 64 \times 240 + \frac{1}{3}\pi \times 64 \times 36\right) \text{cm}^3$

$= \pi \times 64 \times (240 + 12)\text{cm}^3$

$= \frac{22}{7} \times 64 \times 252\text{cm}^3 = 22 \times 64 \times 36\text{cm}^3$

Total weight of the pillar =Volume$\times$Weight per cm^3

$= (22 \times 64 \times 36) \times 7.8\text{gms}$

= 395366.4 gms = 395.3664 kg

Section E

36. i. Child's Day wise are,

$\underbrace{5}_{1\ coin}, \underbrace{10}_{2\ coins}, \underbrace{15}_{3\ coins}, \underbrace{20}_{4\ coins}, \underbrace{25}_{5\ coins}$ .... to $\underbrace{n\ days}_{n\ coins}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n - 1)1]$ = 190

$\Rightarrow$ n(n + 1) = 380 $\Rightarrow$ n^2 + n - 380 = 0

$\Rightarrow$ (n + 20)(n − 19) = 0 $\Rightarrow$ (n + 20)(n - 19) = 0

$\Rightarrow$ n = −20 or n = 19 $\Rightarrow$ n = -20 or n = 19

But number of coins cannot be negative

$\therefore$ n = 19 (rejecting n = -20)

So, number of days = 19

ii. Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

= $\frac{19}{2}$[2 $\times$ 5 + (19 - 1)5]

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹ 950

iii. Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10-1) \times 5]$

$\Rightarrow S_{10}$ = 5[10 + 45}]

$\Rightarrow$ S_{10} = 275

Money saved in 10 days = ₹ 275

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}$ [2 + 14]

$\Rightarrow$ S_{15} = 120

So, there are 120 coins on 15th day.

37. i. The maximum class frequency is 15 belonging to class interval 150-155

$\therefore$ 150 - 155 is the modal class

lower limit (l) of modal class =150

ii.

Height (in cm)	frequency	C.F
135-140	2	2
140-145	8	10
145-150	10	20
150-155	15	35
155-160	6	41
160-165	5	46
165-170	4	50
	$\sum fi$ = 50	

$\sum fi$ = 2 + 8 + 10 + 15 + 6 + 5 + 4 =50 =N

$\frac{N}{2} = \frac{50}{2}$ = 25

c.f just greater that $\frac{N}{2}$ i.e, 25 is 35

$\therefore$ Median class 150-155

iii.

Height (in cm)	frequency (f_i)	x_i
135-140	2	137.5
140-145	8	142.5
145-150	10	147.5
150-155	15	**152.5**
155-160	6	157.5
160-165	5	162.5
165-170	4	167.5

$x_i = \frac{\text{lower limit+upper limit}}{2}$

middle term of x_i,is the assumean mean

Hence, Assumed Mean = 152.5

OR

Median = $l\left(\frac{\frac{n}{2}-c.f}{f}\right) \times h$

= 150 + $\left(\frac{25-20}{15}\right) \times 5$

= 150 + $\frac{5}{15} \times 5$

= 150 + $\frac{5}{3}$

= 150 + 1.67

= 151.67

38. i. x + y = 300 ...(i)

150 x + 250 y = 55000 ...(ii)

ii. a. Solving equation (i) and (ii)

Number of children visited park (x) = 200

OR

b. Solving equation (i) and (ii)

Number of adults visited park (y) = 100

iii. Amount collected = 250 $\times$ 150 + 100 $\times$ 250 = ₹ 62500