Class X Session 2025-26
Subject - Mathematics (Standard)
Sample Question Paper - 09

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are AssertionReason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study based questions carrying 4 marks each with sub parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Question of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E .
9. Draw neat and clean figures wherever required.
10. Take wherever required if not stated.
11. Use of calculators is not allowed.

Section A

1. The HCF of 95 and 152, is **[1]**

a) 57 b) 19

c) 1 d) 38

2. If a is rational and $\sqrt{b}$ is irrational, then $a+\sqrt{b}$ is: **[1]**

a) a rational number b) a natural number

c) an irrational number d) an integer

3. If the sum and the product of zeroes of a quadratic polynomial are $2\sqrt{3}$ and 3 respectively, then a quadratic polynomial is: **[1]**

a) $x^2-2\sqrt{3}x-3$ b) $(x-\sqrt{3})^2$

c) $x^2+2\sqrt{3}x+3$ d) $x^2+2\sqrt{3}x-3$

4. If α and β are the zeroes of the polynomial ax^2 - 5x + c and $\alpha+\beta=\alpha\beta$ = 10, then: **[1]**

a) a = 5, c = $\frac{1}{2}$ | b) a = $\frac{1}{2}$, c = 5

c) a = $\frac{5}{2}$, c = 1 | d) a = 1, c = $\frac{5}{2}$

5. In △ABC, if ∠C = 3∠B = 2(∠A + ∠B), then ∠C = **[1]**

a) 120^o

b) 60^o

c) 150^o

d) 90^o

6. The value of a so that the point (3, a) lies on the line represented by 2x - 3y = 5 is **[1]**

a) $\frac{-1}{3}$

b) $\frac{1}{3}$

c) – 1

d) 1

7. The roots of the quadratic equation $9a^2b^2x^2 - 16abcdx - 25c^2d^2 = 0$ are **[1]**

a) $\frac{-25cd}{9ab}$ and $\frac{-cd}{ab}$

b) $\frac{25cd}{9ab}$ and $\frac{cd}{ab}$

c) $\frac{-25cd}{9ab}$ and $\frac{cd}{ab}$

d) $\frac{25cd}{9ab}$ and $\frac{-cd}{ab}$

8. The discriminant of the quadratic equation $2x^2 + x - 1 = 0$ is: **[1]**

a) 7

b) 9

c) -9

d) -7

9. If 9^{th} term of an A.P. is zero, then its 29^{th} term is ________ its 19^{th} term. **[1]**

a) Equal to

b) Half of

c) Twice of

d) Thrice of

10. What is the common difference of an AP in which $a_{18} - a_{14} = 32$? **[1]**

a) -8

b) 4

c) -4

d) 8

11. In △ABC, D and E are points on side AB and AC respectively such that DE || BC and AD : DB = 3 : 1. If EA = 3.3 cm, then AC = **[1]**

a) 5.5 cm

b) 4.4 cm

c) 4 cm

d) 1.1 cm

12. The diameter of a circle is of length 6 cm. If one end of the diameter is (-4, 0), the other end on x-axis is at: **[1]**

a) (2, 0)

b) (6, 0)

c) (4, 0)

d) (0, 2)

13. $\cot^2\theta - \frac{1}{\sin^2\theta}$ is equal to: **[1]**

a) -1

b) 2

c) 1

d) -2

14. If x tan 45° cos 60° = sin 60° cot 60°, then x is equal to **[1]**

a) $\sqrt{3}$

b) $\frac{1}{\sqrt{2}}$

c) 1

d) $\frac{1}{2}$

15. A circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^o$, if AD = 23 cm, AB = 29 cm and DS = 5 cm, **[1]**

then radius of circle is :

a) 13 cm
b) 12 cm
c) 11 cm
d) 14 cm

16. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by [1]

a) $\frac{4}{9}$cm
b) $\frac{9}{4}$cm
c) $\frac{2}{9}$cm
d) $\frac{9}{2}$cm

17. Consider the following frequency distribution: [1]

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is

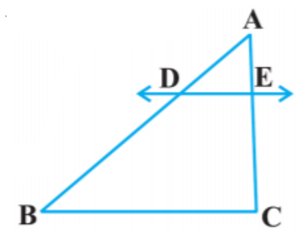
a) 17.5
b) 17
c) 18
d) 18.5

18. Which of the following is not probability of an event? [1]

a) $\frac{1}{13}\%$
b) $\frac{1}{0.89}$
c) 0.89
d) 52%

19. **Assertion (A):** D and E are points on the sides AB and AC respectively of a △ABC such that DE||BC then the value of x is 11, when AD = 4cm, DB = (x - 4)cm, AE = 8cm and EC = (3x - 19)cm. [1]

Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.



a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The point (-1, 6) divides the line segment joining the points (-3, 10) and (6, -8) in the ratio 2 : 7 internally. [1]

Reason (R): Three points A, B and C are collinear if area of △ABC = 0.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468. [2]

22. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction. [2]

23. If the mid-point of the line segment joining the points A (3, 4) and B (k, 6) is P (x, y) and x + y - 10 = 0, find the [2]

value of k.

OR

Write the ratio in which the line segment doming the points A (3, - 6) and B (5, 3) is divided by X-axis.

24. Prove that: $\frac{\sec \theta+\tan \theta}{\sec \theta-\tan \theta}=\left(\frac{1+\sin \theta}{\cos \theta}\right)^2$ **[2]**

OR

Prove the trigonometric identity : $\frac{\sin \theta}{1+\cos \theta}+\frac{1+\cos \theta}{\sin \theta}$ = 2 cosec θ.

25. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is 7? **[2]**

Section C

26. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, then find their monthly incomes. **[3]**

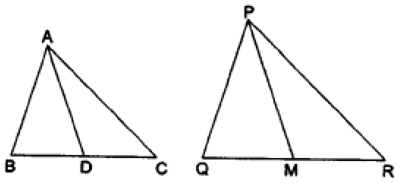
OR

Draw the graph of the pair of equations 2x + y = 4 and 2x – y = 4. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

27. Find the value of p for which the points (-1, 3), (2, p) and (5, -1) are collinear. **[3]**

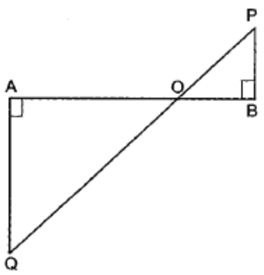
28. If $\frac{\cos \alpha}{\cos \beta}$ = m and $\frac{\cos \alpha}{\sin \beta}$ = n show that $(m^2 + n^2) \cos^2\beta = n^2$ **[3]**

29. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle$PQR (see figure). Show that $\triangle ABC \sim \triangle PQR$. **[3]**

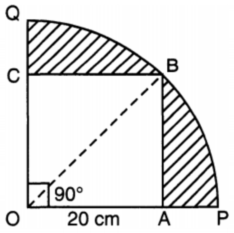


OR

In Fig. QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.



30. A square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region. [Use π = 3.14] **[3]**



31. Prove that the tangents drawn from an external point to a circle are equal in length. **[3]**

Section D

32. A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more, find the speed of the faster and the passenger train. **[5]**

OR

Solve: $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2, x \neq -\frac{1}{2}, 1$

33. In an A.P., the n^{th} term is $\frac{1}{m}$ and the m^{th} term is $\frac{1}{n}$. Find (i) $(mn)^{th}$ term, (ii) sum of first (mn) terms. **[5]**

34. From the top of a building 15 m high, the angle of elevation of the top of a tower is found to be 30^{o}. Form the bottom of the same building, the angle of elevation of the top of the tower is found to be 45^{o}. Determine the height of the tower and the distance between the tower and the building. **[5]**

OR

A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point.

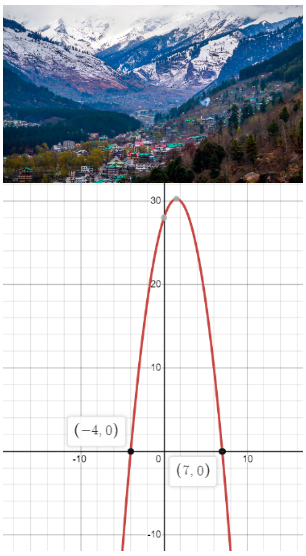
35. The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three methods discussed in this section. **[5]**

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of states/U.T.	6	11	7	4	4	2	1

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Two friends Govind and Pawan decided to go for a trekking. During summer vacation, they went to Panchmarhi. While trekking they observed that the trekking path is in the shape of a parabola. The mathematical representation of the track is shown in the graph.



i. What are the zeroes of the polynomial whose graph is given? (1)
ii. What will be the expression of the given polynomial p(x)? (1)
iii. What is the product of the zeroes of the polynomial which represents the parabola? (2)

OR

In the standard form of quadratic polynomial, $ax^2 + bx + c$, whar are a, b, and c? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Singing bowls (hemispherical in shape) are commonly used in sound healing practices. Mallet (cylindrical in shape) is used to strike the bowl in a sequence to produce sound and vibration.

One such bowl is shown here whose dimensions are:

Hemispherical bowl has outer radius 6 cm and inner radius 5 cm.

Mallet has height of 10 cm and radius 2 cm.

i. What is the volume of the material used in making the mallet? (1)
ii. The bowl is to be polished from inside. Find the inner surface area of the bowl. (1)
iii. Find the volume of metal used to make the bowl. (2)

OR

Find total surface area of the mallet. (Use π = 3.14) (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

i. What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B? (1)
ii. If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is (1)
iii. $7 \times 11 \times 13 \times 15 + 15$ is a (2)

OR

If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q) is (2)

Solution

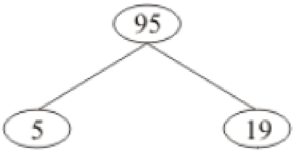
Section A

1.

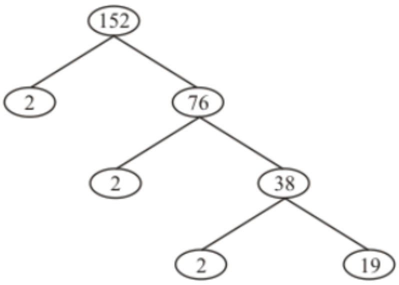
(b) 19

Explanation:

Using the factor tree for 95, we have:



Using the factor tree for 152, we have:



Therefore,

$95 = 5 \times 19$

$152 = 2^3 \times 19$

HCF (95, 152) = 19

2.

(c) an irrational number

Explanation:

Let a be rational and $\sqrt{b}$ is irrational.

If possible let $a + \sqrt{b}$ be rational.

Then $a + \sqrt{b}$ is rational and a is rational.

$\Rightarrow \left[\left(a + \sqrt{b}\right) - a\right]$ is rational [Difference of two rationals is rational]

$\Rightarrow \sqrt{b}$ is rational.

This contradicts the fact that $\sqrt{b}$ is irrational.

The contradiction arises by assuming that $a + \sqrt{b}$ is rational.

Therefore, $a + \sqrt{b}$ is irrational.

3.

(b) $(x - \sqrt{3})^2$

Explanation:

$\alpha + \beta = 2\sqrt{3}$

$\alpha\beta = 3$

required quad. poly is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - (2\sqrt{3})x + 3 = 0$

$x^2 - 2\sqrt{3}x + 3 = 0$

$(x - \sqrt{3})^2 = 0$

4.

(b) a = $\frac{1}{2}$, c = 5

Explanation:

P(x) = x^2 - (sum of roots)x + (product of roots)

x^2 - (10)x + 10 = 0

$\frac{1}{2}x^2$ - 5x + 5 = 0 ...(1)

Given, ax^2 - 5x + c = 0 ...(2)

comparing (1) and (2), we get,

a = $\frac{1}{2}$, c = 5

5. **(a)** 120^o

Explanation:

Since ∠A + ∠B + ∠C = 180^o ... (i)

∠C = 3∠B = 2(∠A+∠B)

3∠B = 2(∠A+∠B)

3∠B - 2∠B = 2∠A

∠B = 2∠A

∠A = $\angle\frac{B}{2}$

from (i),

$\angle\frac{B}{2}$ + ∠B + 3∠B = 180^o

$9\angle\frac{B}{2}$ = 180^o

∠B = 40^o

∠C = 3∠B

∠C = 3 × 40 = 120^o

6.

(b) $\frac{1}{3}$

Explanation:

2x - 3y = 5

⇒ 2 × 3 - 3 × a = 5

⇒ 6 - 3a = 5

⇒ a = $\frac{1}{3}$

7.

(d) $\frac{25cd}{9ab}$ and $\frac{-cd}{ab}$

Explanation:

Using factorisation method

$9a^2b^2x^2$ - 16abcdx - $25c^2d^2$ = 0

⇒ $9a^2b^2x^2$ - 25abcdx + 9abcdx - $25c^2d^2$ = 0

⇒ abx(9abx - 25cd) + cd(9abx - 25cd) = 0

⇒ (abx + cd)(9abx - 25cd) = 0

⇒ abx + cd = 0 and 9abx - 25cd = 0

⇒ x = $\frac{-cd}{ab}$ and x = $\frac{25cd}{9ab}$

8.

(b) 9

Explanation:

D = b^2 - 4ac

Here, a = 2, b = 1 and c = -1

b^2 - 4ac = $(1)^2$ - 4(2)(-1)

= 1 + 8

= 9

9.

(c) Twice of

Explanation:

Let 1st term of A.P. be a and common difference be d.

Now, $a_9 = 0 \Rightarrow a + 8d = 0 \Rightarrow a = -8d$ ...(i)

Now, $a_{29} = a + 28d = -8d + 28d$ ...(ii)

$\Rightarrow a_{29} = 20d$

Also, $a_{19} = a + 18d = -8d + 18d = 10d$

$\Rightarrow 2 \times a_{19} = 2 \times 10d = 20d$ ...(iii)

From (ii) and (iii), we have

$a_{29} = 2\ a_{19}$

10.

(d) 8

Explanation:

$a_{18} - a_{14} = 32$

(18 - 14)d = 32

$\Rightarrow$ 4d = 32

$\Rightarrow$ d = 8.

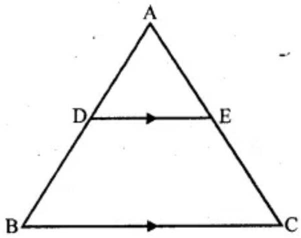
11.

(b) 4.4 cm

Explanation:

In ΔABC, DE || BC

AD : DB = 3 : 1, EA = 3.3 cm



Let EC = x

$\because$ In $\triangle$ABC, DE || BC

$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}} \Rightarrow \frac{3}{1} = \frac{3.3}{x}$

$\Rightarrow x = \frac{3.3}{3} = 1.1\text{cm}$

$\therefore$ AC = AE + EC = 3.3 + 1.1 = 4.4 cm

12. **(a)** (2, 0)

Explanation:

Let the other point on x axis is (x, 0)

By distance formula

$\sqrt{(x+4)^2 + (0-0)^2} = (6)$

x + 4 = 6

x = 2

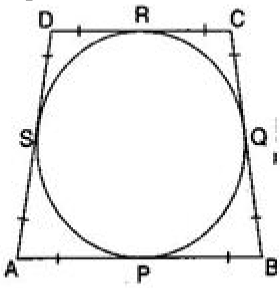
Hence the point is (2, 0)

13. **(a)** -1

Explanation:

Given: $\cot^2\theta$ - $\frac{1}{\sin^2\theta}$
= $\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$
= $\frac{\cos^2\theta - 1}{\sin^2\theta}$
= $\frac{-\sin^2\theta}{\sin^2\theta}$ = -1
[$\because$ 1 - $\cos^2\theta$ = $\sin^2\theta$]

14.
(c) 1
Explanation:
We have, x tan 45° cos 60° = sin 60° cot 60°
$\Rightarrow x \times 1 \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{2} = \frac{1}{2}$
$\Rightarrow x = \frac{1}{2} \times 2 = 1$

15.
(c) 11 cm
Explanation:


Here DS = DR = 5 cm
$\Rightarrow$ AS = 23 – 5 = 18 cm
And AS = AP = 18 cm
$\Rightarrow$BP = 29 – 18 = 11 cm
$\because$OP$\perp$ AB and OQ $\perp$ BC
$\therefore\angle$OQB = $\angle$OPB = 90° and $\angle$B = 90°
Also $\angle$POQ = 90°
Therefore, OPBQ is a square.
$\therefore$ BQ = OQ = 11 cm
Therefore Radius of circle = 11 cm

16.
(b) $\frac{9}{4}$cm
Explanation:
Radius of sphere (r_1) = 3 cm
$\therefore$ Volume = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 \text{cm}^3$
= 36π cm^3
$\therefore$ Volume of water in the cylinder = 36π cm^3
Radius of cylindrical vessel (r_2) = 4 cm
Let h be its height, then
$\pi r_2^2 h = 36\pi \Rightarrow \pi(4)^2 h = 36\pi$
$\Rightarrow 16\pi h = 36\pi \Rightarrow h = \frac{36\pi}{16\pi} = \frac{9}{4}$cm

17. **(a)** 17.5
Explanation:
Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2}=\frac{57}{2}=28.5$, which lies in the interval 11.5 - 17.5.

Hence, the upper limit is 17.5.

18.

(b) $\frac{1}{0.89}$

Explanation:

$\frac{1}{0.89}$

Probability of an event can never be greater than 1.

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side. This is the Converse of the Basic Proportionality theorem.

So, the Reason is correct.

By Basic Proportionality theorem, we have $\frac{AD}{DB}=\frac{AE}{EC} \Rightarrow \frac{4}{x-4}=\frac{8}{3x-19}$

$\Rightarrow$ 4(3x - 19) = 8(x - 4)

$\Rightarrow$ 12x - 76 = 8x - 32

$\Rightarrow$ 4x = 44 $\Rightarrow$ x = 11 cm

So, Assertion is correct.

But reason (R) is not the correct explanation of assertion (A).

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Using section formula, we have

-1 = $\frac{k\times 6+1\times(-3)}{k+1}$

-k - 1 = 6k - 3

7k = 2

k = $\frac{2}{7}$

Ratio is 2 : 7 internally.

Also, if ar($\triangle$ABC) = 0

A, B and C all these points are collinear.

Section B

21. The smallest number divisible by 520 and 468 = LCM(520,468)

Prime factors of 520 and 468 are :

520 = $2^3 \times 5 \times 13$

468 = $2 \times 2 \times 3 \times 3 \times 13$

Hence LCM(520,468) = $2^3 \times 3^2 \times 5 \times 13$ = 8 $\times$ 9$\times$5$\times$13 = 4680

Now the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

=LCM(520,468)-17

=4680-17

=4663

22. Let the numerator and denominator of fraction be x and y respectively.

Then, the fraction is $\frac{x}{y}$.

As per first condition

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator.

x + y = 2x + 4

⇒ -x + y = 4........(i)

According to the second condition,

If the numerator and denominator are increased by 3, they are in the ratio 2 : 3.

$\frac{x+3}{y+3} = \frac{2}{3}$

⇒ 3x + 9 = 2y + 6

⇒3x - 2y = -3.......(ii)

Multiply (i) by -2, we get

-2x + 2y = 8 .... (iii)

Adding (ii) and (iii) , we get

and 3x - 2x = -3 + 8

⇒ x = 5

Substituting x = 5 in (i), we get

5 - y = 4

y = 9

Hence, the required fraction is $\frac{5}{9}$

23. Mid-point of the line segment joining A(3, 4) and B(k, 6) = $\frac{3+k}{2}$, $\frac{4+6}{2}$

= $\frac{3+k}{2}, 5$

Then, $\frac{3+k}{2}, 5$ = (x, y)

Therefore, $\frac{3+k}{2}$ = x and 5 = y

Since x + y - 10 = 0, we have

$\frac{3+k}{2}$ + 5 - 10 = 0

i.e., 3 + k = 10

Therefore, k = 7

OR

The point lies on x-axis

Its ordinate will be = 0

Let the point P(x, 0) divides the line-segment joining the points A(3, -6) and B(5, 3) in the ratio m:n.

$\therefore 0 = \frac{my_2+ny_1}{m+n} \Rightarrow 0 = \frac{m\times 3+n(-6)}{m+n}$

$\Rightarrow 3m - 6n = 0 \Rightarrow 3m = 6n$

$\Rightarrow \frac{m}{n} = \frac{6}{3} = \frac{2}{1}$

$\therefore$ Ratio = 2:1

24. $\mathrm{L.H.S.} = \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

Multiplying and dividing by $(\sec\theta + \tan\theta)$

$= \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} \times \frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

$= \frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta} \left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= \frac{(\sec\theta+\tan\theta)^2}{1} \left[\because \sec^2\theta - \tan^2\theta = 1\right]$

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$

$\therefore$ L.H.S. $= \mathrm{R.H.S.}$

OR

LHS = $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

= $\frac{\sin\theta\times\sin\theta}{(1+\cos\theta)\times\sin\theta}$ + $\frac{(1+\cos\theta)\times(1+\cos\theta)}{\sin\theta\times(1+\cos\theta)}$

= $\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta\times(1+\cos\theta)}$

Using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$

$= \frac{2+2\cos\theta}{\sin\theta\times(1+\cos\theta)}$

$=\frac{2(1+\cos\theta)}{\sin\theta\times(1+\cos\theta)}$

$=\frac{2}{sin\theta}$

Now $\frac{1}{sin\theta}$ = cosec θ

= 2 cosec θ.

25. Two dice are thrown simultaneously. [given]

So, that number of possible outcomes = 36

Sum of the numbers appearing on the dice is 7.

So, the possible ways are (1, 6), (2,5), (3, 4), (4, 3), (5, 2) and (6, 1).

Number of possible ways = 6

$\therefore$ Required probability = $\frac{6}{36} = \frac{1}{6}$

Section C

26. Let us denote the incomes of the two-person by ₹ 9x and ₹ 7x and their expenditures by ₹ 4y and ₹ 3y respectively.

Then the equations formed in the situation is given by :

9x – 4y = 2000 ...(i)

and 7x – 3y = 2000 ...(2)

Step 1: Multiply Equation (1) by 3 and Equation (2) by 4 to make the coefficients of y equal. Then, we get the equations:

27x – 12y = 6000 ...(3)

28x – 12y = 8000 ...(4)

Step 2: Subtract Equation (3) from Equation (4) to eliminate y, because the coefficients of y are the same. So, we get

(28x – 27x) – (12y – 12y) = 8000 – 6000

i.e., x = 2000

Step 3: Substituting this value of x in (1), we get

9(2000) – 4y = 2000

i.e., y = 4000

So, the solution of the equations is x = 2000, y = 4000. Therefore, the monthly incomes of the persons are ₹18,000 and ₹14,000 respectively.

OR

The given pair of linear equations

2x + y = 4

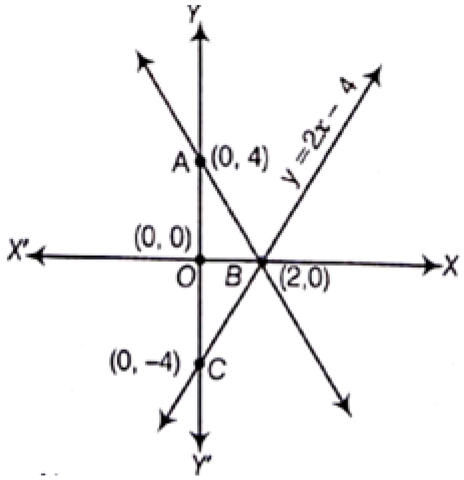
and 2x-y = 4

Table for line 2x + y = 4

x	0	2
y	4	0
Points	A	B

and table for line 2x - y = 4

x	0	2
y	- 4	0
Points	C	B



So the Graphical representation of both lines is as above.

Here, both lines and Y - axis from a $\triangle \mathrm{ABC}$.

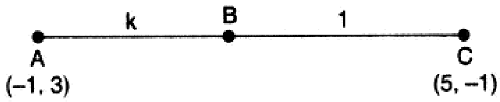
Hence, the vertices of a $\triangle \mathrm{ABC}$ are A (0,4), B(2,0) and C(0,- 4) where A and C are obtained by putting x = 0 in the given equations abd B is obtained by solving them together.

$\therefore$ Required area of $\triangle \mathrm{ABC}$= 2 × Area of $\triangle \mathrm{AOB}$

$\triangle \mathrm{ABC}$ = 2 × $\left(\frac{1}{2} \times 4 \times 2\right) = 8$ sq. units.

Hence, the required area of the triangle is 8 sq units.

27. Let A → (–1, 3) B → (2, p) and C → (5, –1)



If the points A, B and C are collinear then let B divide AC in the ratio K : 1 internally.

Then, $B \rightarrow \left\{\frac{(\mathrm{K})(5)+(1)(-1)}{\mathrm{K}+1}, \frac{(\mathrm{K})(-1)+(1)(3)}{\mathrm{K}+1}\right\}$

$\Rightarrow \mathrm{B} \rightarrow \left(\frac{5\mathrm{K}-1}{\mathrm{K}+1}, \frac{-\mathrm{K}+3}{\mathrm{K}+1}\right)$

But, B is given to be (2,p)

$\therefore \frac{5\mathrm{K}-1}{\mathrm{K}+1} = 2$

$\Rightarrow$ 5K - 1 = 2(K + 1)

$\Rightarrow$ 5K - 1 = 2K + 2

$\Rightarrow$5K - 2K = 2 + 1

$\Rightarrow$3K = 3

$\Rightarrow \mathrm{K} = \frac{3}{3} = 1$ and $\frac{-\mathrm{K}+3}{\mathrm{K}+1} = \mathrm{p}$

$\Rightarrow \frac{-1+3}{1+1} = p$

$\Rightarrow$ p = 1

Hence, the required value of p is 1.

28. Given, m = $\frac{\cos\alpha}{\cos\beta}$ ......(1) and, n = $\frac{\cos\alpha}{\sin\beta}$ ....(2)

LHS = $(m^2 + n^2) \cos^2\beta$

$\Rightarrow \mathrm{LHS} = \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ [from (1) & (2)]

$\Rightarrow \mathrm{LHS} = \left(\frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

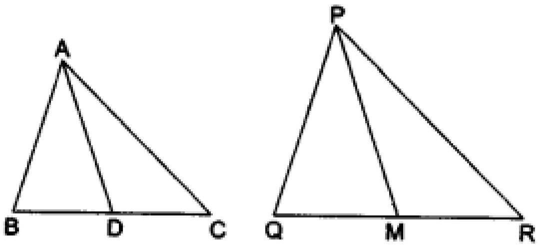
$\Rightarrow \mathrm{LHS} = \cos^2\alpha\left(\frac{\sin^2\beta + \cos^2\beta}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$

$\Rightarrow \mathrm{LHS} = \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$ [Since, $\sin^2 A + \cos^2 A = 1$]

$\Rightarrow L.H.S. = \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta}\right)^2$

$\therefore L.H.S. = n^2$ [from (2)]

= R.H.S . Hence, Proved.

29. 

It is given that:

$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$

$\Rightarrow \quad \frac{AB}{PQ}=\frac{AD}{PM}=\frac{BC}{QR}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}=\frac{BD}{QM}$ ....(i)

In $\triangle ABD$ and $\triangle PQM$, we have

$\frac{AB}{PQ}=\frac{AD}{PM}=\frac{BD}{QM}$ [from(i)]

$\therefore \quad \triangle ABD \sim \triangle PQM$ [by SSS-similarity criteria].

And also, $\angle B=\angle Q$ [corresponding angles of similar triangles are equal].

Now, in $\triangle ABC$ and $\triangle PQR$, we have

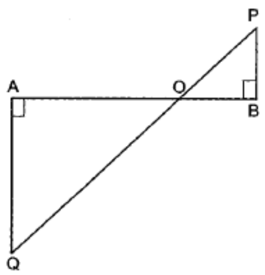
$\angle B=\angle Q$ [proved above]

and $\frac{AB}{PQ}=\frac{BD}{QM}$ [from(i)].

$\therefore \quad \triangle ABC \sim \triangle PQR$ [by SAS-similarity criteria].

OR

In triangles AOQ and BOP, we have



$\angle$OAQ = $\angle$OBP [Each equal to 90°]

$\angle$AOQ = $\angle$BOP [Vertically opposite angles]

Therefore, by AA- criterion of similarity, we obtain

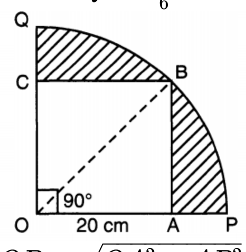
$\Delta AOQ \sim \Delta BOP$

$\Rightarrow \quad \frac{AO}{BO}=\frac{OQ}{OP}=\frac{AQ}{BP}$

$\Rightarrow \quad \frac{AO}{BO}=\frac{AQ}{BP}$

$\Rightarrow \quad \frac{10}{6}=\frac{AQ}{9}$

$\Rightarrow \quad AQ=\frac{10\times 9}{6}=15\mathrm{cm}$

30. 

$OB=\sqrt{OA^2+AB^2}$

$=\sqrt{20^2+20^2}$

$=\sqrt{400+400}$

$=\sqrt{800}$

$=\sqrt{400\times 2}$

$OB=20\sqrt{2}$ cm or, radius = $20\sqrt{2}$

Area of shaded region = Area of sector OQBPO - Area of square OABC

$=\frac{90^{\circ}}{360^{\circ}}\times 3.14\times 20\sqrt{2}\cdot 20\sqrt{2}-(20)^2$

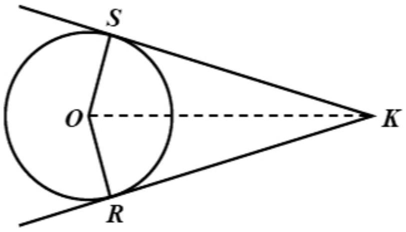
$=\frac{1}{4}\times 3\cdot 14\times 800-400$

$=2(314)-400$

$= 628 - 400$
$= 228$

Required Area = 228 cm^2.

31. The attached figure shows two tangents, SK and SR drawn to circle with center O from an external point K.



To prove that: SK = RK
Proof:
Normal and tangent at a point on the circle are perpendicular to each other.
$\angle OSK = \angle ORK = 90^o$
Using Pythagoras Theorem,
$OK^2 = OS^2 + SK^2$ ...(i)
$OK^2 = OR^2 + RK^2$ ...(ii)
Subtracting (ii) from (i),
$OK^2 - OK^2 = OS^2 + SK^2 - OR^2 - RK^2$
$\Rightarrow SK^2 = RK^2 \because OS = OR$
SK = RK

Section D

32. 

Let speed of passenger train be x km/h
$\therefore$ speed of superfast train = (x + 16) km/h
By question, $T_{\text{passenger}} = \frac{192}{x}$ and $T_{\text{superfast}} = \frac{192}{(x+16)}$
or, $\frac{192}{x} - \frac{192}{x+16} = 2$
or, $192(x+16) - 192x = 2\left(x^2 + 16x\right)$
or, $192x + 192 \times 16 - 192x = 2\left(x^2 + 16x\right)$
192x + 3072 - 192x $= 2\left(x^2 + 16x\right)$ (divide throughout by 2, we get,
96x + 1536 - 96x $= \left(x^2 + 16x\right)$
or x(x + 48) - 32(x + 48) = 0
or, (x - 32) (x + 48) = 0
or, x = 32 or - 48
Since speed can't be negative, therefore - 48 is not possible.
$\therefore$ Speed of passenger train = 32 km/h and Speed of fast train = 48 km/h

OR

Given
$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$
Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$
$\therefore$ Substituting this value
$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$
or $y^2 + 1 = 2y$
or $y^2 - 2y + 1 = 0$
or $(y-1)^2 = 0$
Putting $y = \frac{x-1}{2x+1}$,
$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$
or x = -2

33. $a_n = \frac{1}{m}$

$a + (n - 1)d = \frac{1}{m}$

$a_m = \frac{1}{n}$

$a + (m - 1)d = \frac{1}{n}$

On solving,

$a = \frac{1}{mn}$

$d = \frac{1}{mn}$

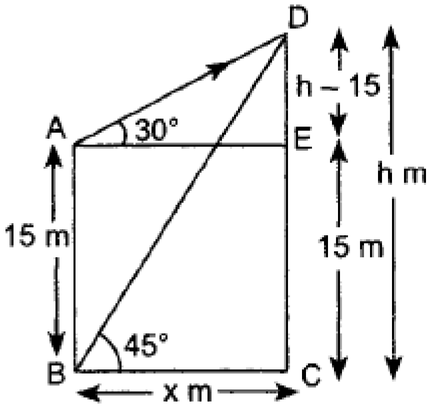
i. $a_{mn} = \frac{1}{mn} + (mn - 1) \times \frac{1}{mn}$

$= \frac{1+mn-1}{mn} = 1$

ii. $S_{mn} = \frac{mn}{2}\left(\frac{1}{mn} + 1\right)$

$= \frac{1+mn}{2}$

34. According to question it is given that a building AB of height 15 m and tower CD of h meter respectively.



Angle of elevation $\angle DAE = 30^o$

Angle of elevation $\angle DBC = 45^o$

To find: BC and CD

Proof: In right $\triangle DEA$, using Pythagoras theorem

$\frac{DE}{x} = \tan 30^\circ$ ($\because$ AE = BC = x)

$\Rightarrow \quad \frac{h-15}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad h - 15 = \frac{x}{\sqrt{3}}$ ....(i)

In right $\triangle DCB$, by Pythagoras theorem

$\frac{h}{x} = \tan 45^\circ$

$\Rightarrow \quad h = x$ .....(ii)

Putting the value of h in equation (i)

$x-15 = \frac{x}{\sqrt{3}}$ [from (i)]

$\Rightarrow \quad 15 = x - \frac{x}{\sqrt{3}}$

$\Rightarrow \quad 15 = \left(1 - \frac{\sqrt{3}}{3}\right) x$

$\Rightarrow \quad 15 = \left(\frac{3-\sqrt{3}}{3}\right) x$

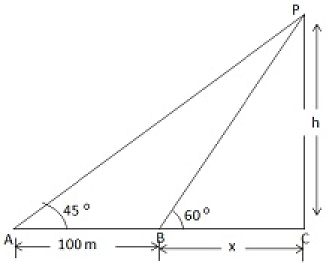
$\Rightarrow \quad 45 = (3 - 1.732)x$

$\Rightarrow \frac{45}{1.268} = x$

x = 35.49

Thus h = 35.49 m

OR



Let PC be the height *h* of the parachutist and makes an angle of elevations between 45° and 60°respectively at two observing points 100m apart from each other

Let AB = 100

Let distance (BC) = x m

In ΔPBC

$\tan 60^o = \frac{PC}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ ...(i)

In ΔPAC

$\tan 45^o = \frac{PC}{AC}$

$\Rightarrow 1 = \frac{h}{100+x}$

$\Rightarrow h = 100 + x$

$\Rightarrow h = 100 + \frac{h}{\sqrt{3}}$ [From (1)]

$\Rightarrow \sqrt{3}h = 100\sqrt{3} + h$ [Multiply by $\sqrt{3}$]

$\Rightarrow \sqrt{3}h - h = 100\sqrt{3}$

$\Rightarrow h\left(\sqrt{3} - 1\right) = 100\sqrt{3}$

$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ ...(2)

$\Rightarrow h = \frac{100(3+\sqrt{3})}{3-1}$

$\Rightarrow h = 50\left(3 + 1.732\right)$

$\Rightarrow h = 50 \times 4.732 = 236.6m$

Put value of h from equation (2) in equation (1)

$x = \frac{1}{\sqrt{3}} \times \frac{100\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow x = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow x = \frac{1000(1.732+1)}{3-1}$

$\Rightarrow x = \frac{1000 \times 2.732}{2} = 100 \times 1.366 = 136.6m$

$\therefore$ Height of parachutist = 236.6m

Distance of first point from Where he falls = 136.6 m

35. Let, $a = 50$

C.I.	**Number of states/ U.T. (f_i)**	**x_i**	**$d_i = x_i - 50$**	**f_id_i**
15 - 25	6	20	-30	-180
25 - 35	11	30	-20	-220
35 - 45	7	40	-10	-70
45 - 55	4	50	0	0
55 - 65	4	60	10	40
65 - 75	2	70	20	40
75 - 85	1	80	30	30

From table, $\Sigma f_i d_i = -360, \Sigma f_i = 36$

we know that, mean=$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 50 + \frac{-360}{35}$
$= 39.71$

Section E

36. i. Point of intersection of graph of polynomial, gives the zeroes of the polynomial.
$\therefore$ zeroes = -4 and 7
ii. Since, zero's are α = -4, β = 7
$\alpha + \beta$ = -4 + 7 = 3
$\alpha\beta$ = -4 $\times$ 7 = -28
P(x) = x^2 - (Sum of zeroes)x + product of zeroes
P(x) = x^2 - 3x + (-28)
P(x) = x^2 - 3x - 28
iii. Product of zeroes = -4 $\times$ 7
= -28
OR
a is a non-zero real number, b and c are any real numbers c.
37. i. mallet is cylindrical in shape.
Volume = $\pi \times r^2 \times$ h
= $\pi \times (2\text{ cm})^2 \times$ 10cm
= $\pi \times 4\text{ cm}^2 \times$ 10 cm
= $40\pi\text{ cm}^3$
ii. Inner Surface Area = $2\pi \times (5\text{ cm})^2$
Inner Surface Area = $2\pi \times 25\text{ cm}^2$
Inner Surface Area = $50\pi\text{ cm}^2$
iii. Outer Hemisphere Volume = $(\frac{2}{3})\pi\ (6\text{ cm})^3$
= $(\frac{2}{3})\pi\ (216\text{ cm}^3)$
$\approx 144\pi\text{ cm}^3$
Inner Hemisphere Volume = $(\frac{2}{3})\pi\ (5\text{ cm})^3$
= $(\frac{2}{3})\pi\ (125\text{ cm}^3)$
$\approx \frac{250\pi}{3}\text{ cm}^3$
Volume of Metal Used = Outer Hemisphere Volume - Inner Hemisphere Volume
= $(144\pi\text{ cm}^3)$ - $(\frac{250\pi}{3}\text{ cm}^3)$
= $(\frac{432\pi}{3} - \frac{250\pi}{3})\text{ cm}^3$
= $\frac{182\pi}{3}\text{ cm}^3$
OR
Lateral Surface Area = 2πrh
= 2 $\times$ 3.14 $\times$ 2 cm $\times$ 10 cm
= 125.6 cm^2
For the top circular end:
Area = 3.14 $\times (2\text{ cm})^2$ = 12.56 cm^2
For the bottom circular end (which is the same as the top):
Area = 12.56 cm^2
Total Surface Area = Lateral Surface Area + 2 $\times$ Circular End Areas
= 125.6 cm^2 + 2 $\times$ 12.56 cm^2
= 125.6 cm^2 + 25.12 cm^2
= 150.72 cm^2
So, the total surface area of the mallet is 150.72 square centimeters.

38. i. The number of students in Section A is 32, and the number of students in Section B is 36.

Step 1: Find the prime factors of each number:

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$36 = 2 \times 2 \times 3 \times 3$

Step 2: Identify the common and uncommon prime factors. The common ones are 2×2.

Step 3: Multiply the common and uncommon prime factors together to get the LCM:

$LCM = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$

So, the minimum number of books needed to be acquired for the class library is 288, so they can be distributed equally among students of Section A or Section B.

ii. Step 1: Find the prime factors of each number:

$32 = 2 \times 2 \times 2 \times 2$

$36 = 2 \times 2 \times 3 \times 3$

Step 2: Identify the common prime factors and their minimum exponent:

The common prime factors are 2×2.

Step 3: Calculate the HCF by multiplying the common prime factors:

$HCF = 2 \times 2 = 4$

So, the HCF of 32 and 36 is 4.

iii. Given number $(7 \times 11 \times 13 \times 15 + 15)$

It can also be written as $15 (7 \times 11 \times 13 +1)$.

As it is a product of two composite numbers

hence it is a composite number.

OR

Given:

$p = ab^2$

$q = a^2b$

Take the highest power of each prime factor:

$LCM = a^2 \times b^2$

So, the LCM of p and q is a^2b^2.