रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

नोट

(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं ।

(II) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं ।

(III) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**

(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

NOTE

(I) Please check that this question paper contains **23** printed pages.

(II) Please check that this question paper contains **38** questions.

(III) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(IV) **Please write down the serial number of the question in the answer-book at the given place before attempting it.**

(V) 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

गणित (मानक)

MATHEMATICS (Standard)

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 80
Maximum Marks : 80

● ● ●

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका पालन कीजिए :

(i)   इस प्रश्न-पत्र में 38 प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।

(ii)   यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।

(iii) खण्ड – **क** में प्रश्न संख्या 1 से 18 तक बहुविकल्पीय प्रश्न (MCQs) तथा प्रश्न संख्या 19 एवं 20 अभिकथन एवं तर्क आधारित 1 अंक के प्रश्न हैं ।

(iv)   खण्ड – **ख** में प्रश्न संख्या 21 से 25 तक अति लघु-उत्तरीय (VSA) प्रकार के 2 अंकों के प्रश्न हैं ।

(v)   खण्ड – **ग** में प्रश्न संख्या 26 से 31 तक लघु-उत्तरीय (SA) प्रकार के 3 अंकों के प्रश्न हैं ।

(vi)   खण्ड – **घ** में प्रश्न संख्या 32 से 35 तक दीर्घ-उत्तरीय (LA) प्रकार के 5 अंकों के प्रश्न हैं ।

(vii) खण्ड – **ङ** में प्रश्न संख्या 36 से 38 तक प्रकरण अध्ययन आधारित 4 अंकों के प्रश्न हैं । प्रत्येक प्रकरण अध्ययन में आंतरिक विकल्प 2 अंकों के प्रश्न में दिया गया है ।

(viii) प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड – **ख** के 2 प्रश्नों में, खण्ड – **ग** के 2 प्रश्नों में, खण्ड – **घ** के 2 प्रश्नों में तथा खण्ड-**ङ** के 2 अंकों के 3 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।

(ix)   जहाँ आवश्यक हो, स्वच्छ आकृतियाँ बनाइए । यदि आवश्यक हो तो $\pi = \dfrac{22}{7}$ लीजिए, यदि अन्यथा न दिया गया हो ।

(x)   कैल्कुलेटर का उपयोग **वर्जित** है ।

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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains **38** questions. **All** questions are **compulsory**.

(ii) This question paper is divided into **FIVE** Sections – **A, B, C, D** and **E**.

(iii) In Section–**A** question numbers **1** to **18** are Multiple Choice Questions (MCQs) and question numbers **19** and **20** are Assertion-Reason based questions of **1** mark each.

(iv) In Section–**B** question numbers **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.

(v) In Section–**C** question numbers **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In Section–**D** question numbers **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.

(vii) In Section–**E** question numbers **36** to **38** are **Case Study based integrated** question carrying **4** marks each. Internal choice is provided in **2** marks question in each case-study.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section–**B**, **2** questions in Section–**C**, **2** questions in Section–**D** and **3** questions of **2** marks in Section–**E**.

(ix) Draw neat diagrams wherever required. Take $\pi = \dfrac{22}{7}$ wherever required, if not stated.

(x) Use of calculators is **NOT** allowed.

खण्ड – क

इस खण्ड में 20 बहुविकल्पीय प्रश्न हैं । प्रत्येक प्रश्न का 1 अंक है ।

1. $\sqrt{0.4}$ एक

   (A) प्राकृत संख्या है । (B) पूर्णांक है ।
   (C) परिमेय संख्या है । (D) अपरिमेय संख्या है ।

   1

2. यदि n एक प्राकृत संख्या है, तो 8^n का इकाई का अंक कभी भी नहीं हो सकता है :

   (A) 4 (B) 2
   (C) 0 (D) 6

   1

3. निम्न में से किस द्विघात समीकरण के मूल वास्तविक और बराबर हैं ?

   (A) $(x + 1)^2 = 2x + 1$ (B) $x^2 + x = 0$
   (C) $x^2 - 4 = 0$ (D) $x^2 + x + 1 = 0$

   1

4. यदि बहुपद $ax^2 + bx + \dfrac{2a}{b}$ के शून्यक एक–दूसरे के व्युत्क्रम हैं, तो b का मान है

   (A) 2 (B) $\dfrac{1}{2}$

   (C) −2 (D) $-\dfrac{1}{2}$

   1

5. x-अक्ष से बिंदु A(−3, −4) की दूरी है

   (A) 3 (B) 4
   (C) 5 (D) 7

   1

6. दी गई आकृति में, PQ ∥ XY ∥ BC, AP = 2 cm, PX = 1.5 cm और BX = 4 cm है । यदि QY = 0.75 cm है, तो AQ + CY बराबर है

   1
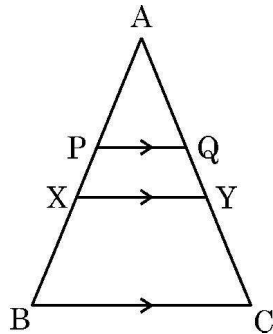
   (A) 6 cm (B) 4.5 cm
   (C) 3 cm (D) 5.25 cm

• • •

## SECTION – A

This section consists of 20 multiple choice questions of 1 mark each.

1. $\sqrt{0.4}$ is a/an   1
   (A) natural number   (B) integer
   (C) rational number   (D) irrational number

2. Which of the following cannot be the unit digit of 8^n, where n is a natural number ?   1
   (A) 4   (B) 2
   (C) 0   (D) 6

3. Which of the following quadratic equations has real and equal roots ?   1
   (A) $(x + 1)^2 = 2x + 1$ (B) $x^2 + x = 0$
   (C) $x^2 - 4 = 0$ (D) $x^2 + x + 1 = 0$

4. If the zeroes of the polynomial $ax^2 + bx + \dfrac{2a}{b}$ are reciprocal of each other, then the value of b is   1

   (A) 2   (B) $\dfrac{1}{2}$

   (C) -2 (D) $-\dfrac{1}{2}$

5. The distance of the point A(-3, -4) from x-axis is   1
   (A) 3   (B) 4
   (C) 5   (D) 7

6. In the adjoining figure, PQ ∥ XY ∥ BC, AP = 2 cm, PX = 1.5 cm and BX = 4 cm. If QY = 0.75 cm, then AQ + CY =   1
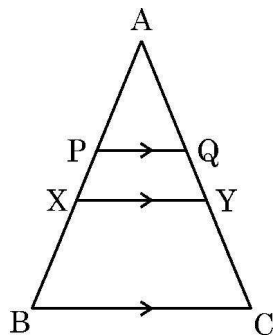
   (A) 6 cm   (B) 4.5 cm
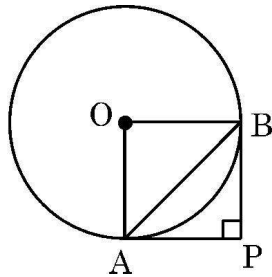   (C) 3 cm   (D) 5.25 cm

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7. दिया गया है कि ΔABC ~ ΔPQR, ∠A = 30° और ∠Q = 90° । (∠R + ∠B) का माप है   **1**

(A) 90°   (B) 120°

(C) 150°   (D) 180°

8. दो सिक्कों को एक साथ उछाला जाता है । कम से कम एक चित आने की प्रायिकता है :   **1**

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{3}{4}$ (D) 1

9. दी गई आकृति में, केन्द्र O वाले एक वृत्त पर PA और PB दो स्पर्श-रेखाएँ इस प्रकार बनी हैं कि ∠P = 90° । यदि AB = $3\sqrt{2}$ cm है, तो वृत्त के व्यास की लंबाई है :   **1**

(A) $3\sqrt{2}$ cm   (B) $6\sqrt{2}$ cm

(C) 3 cm   (D) 6 cm

10. केन्द्र O और त्रिज्या 5 cm वाले एक वृत्त के लिए निम्न कथनों में से कौन सा सही है ?   **1**

P : प्रत्येक समांतर स्पर्श-रेखाओं के युग्म के बीच की दूरी 5 cm है ।

Q : प्रत्येक समांतर स्पर्श-रेखाओं के युग्म के बीच की दूरी 10 cm है ।

R : प्रत्येक समांतर स्पर्श-रेखाओं के युग्म के बीच की दूरी 5 cm और 10 cm के बीच होती है ।

S : वृत्त के बाहर ऐसा कोई बिंदु नहीं होता जिससे वृत्त पर डाली गई स्पर्श-रेखा की लंबाई 5 cm हो ।

(A) P   (B) Q
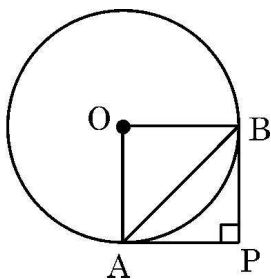
(C) R   (D) S

• • •

7. Given ΔABC ~ ΔPQR, ∠A = 30° and ∠Q = 90°. The value of (∠R + ∠B) is   **1**

(A) 90°   (B) 120°

(C) 150°   (D) 180°

8. Two coins are tossed simultaneously. The probability of getting atleast one head is   **1**

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{3}{4}$ (D) 1

9. In the adjoining figure, PA and PB are tangents to a circle with centre O such that ∠P = 90°. If AB = $3\sqrt{2}$ cm, then the diameter of the circle is   **1**

(A) $3\sqrt{2}$ cm   (B) $6\sqrt{2}$ cm

(C) 3 cm   (D) 6 cm

10. For a circle with centre O and radius 5 cm, which of the following statements is true ?   **1**

P : Distance between every pair of parallel tangents is 5 cm.

Q : Distance between every pair of parallel tangents is 10 cm.

R : Distance between every pair of parallel tangents must be between 5 cm and 10 cm.

S : There does not exist a point outside the circle from where length of tangent is 5 cm.
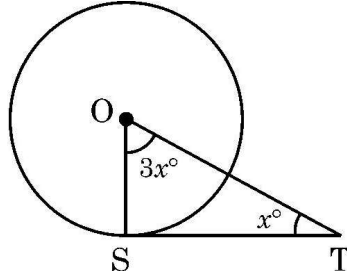
(A) P   (B) Q

(C) R   (D) S

●●●

11. दी गई आकृति में, केन्द्र O वाले एक वृत्त पर TS एक स्पर्श-रेखा है । $2x°$ का मान है   **1**

   (A) 22.5   (B) 45
   (C) 67.5   (D) 90

12. यदि $x\left(\dfrac{2\tan 30°}{1 + \tan^2 30°}\right) = y\left(\dfrac{2\tan 30°}{1 - \tan^2 30°}\right)$ है, तो $x : y$ है   **1**

   (A) 1 : 1   (B) 1 : 2
   (C) 2 : 1   (D) 4 : 1

13. 10 m ऊँचाई के एक पेड़ के शीर्ष पर बैठा एक मोर एक साँप को जमीन पर चलते हुए देखता है । यदि साँप पेड़ के पाद से $10\sqrt{3}$ m की दूरी पर हो, तो मोर की आँख से साँप का अवनमन कोण है   **1**

   (A) 30°   (B) 45°
   (C) 60°   (D) 90°

14. यदि ठोस लकड़ी के एक बेलन से अधिकतम संभव आयतन का एक शंकु काट कर निकाला जाए, तो बची लकड़ी के आयतन का निकाले गए शंकु के आयतन से अनुपात है   **1**

   (A) 1 : 1   (B) 1 : 3
   (C) 2 : 1   (D) 3 : 1

15. यदि कुछ आँकड़ों का बहुलक 10 और माध्य और माध्यक का योग 25 है, तो उन आँकड़ों के माध्य और माध्यक क्रमशः हैं   **1**

   (A) 12 और 13   (B) 13 और 12
   (C) 10 और 15   (D) 15 और 10

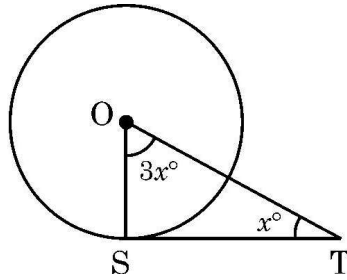16. यदि अधिकतम विद्यार्थियों ने 80 में से 52 अंक प्राप्त किए हों, तो   **1**

   (A) 52 इन आँकड़ों का माध्य है ।   (B) 52 इन आँकड़ों का माध्यक है ।
   (C) 52 इन आँकड़ों का बहुलक है ।   (D) 52 इन आँकड़ों की सीमा है ।

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11. In the adjoining figure, TS is a tangent to a circle with centre O. The value of $2x°$ is   **1**

(A) 22.5

(B) 45

(C) 67.5

(D) 90

12. If $x\left(\dfrac{2\tan 30°}{1+\tan^2 30°}\right)=y\left(\dfrac{2\tan 30°}{1-\tan^2 30°}\right)$, then $x:y=$ **1**

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 4 : 1

13. A peacock sitting on the top of a tree of height 10 m observes a snake moving on the ground. If the snake is $10\sqrt{3}$ m away from the base of the tree, then angle of depression of the snake from the eye of the peacock is   **1**

(A) 30°

(B) 45°

(C) 60°

(D) 90°

14. If a cone of greatest possible volume is hollowed out from a solid wooden cylinder, then the ratio of the volume of remaining wood to the volume of cone hollowed out is   **1**

(A) 1 : 1

(B) 1 : 3

(C) 2 : 1

(D) 3 : 1

15. If the mode of some observations is 10 and sum of mean and median is 25, then the mean and median respectively are   **1**

(A) 12 and 13

(B) 13 and 12

(C) 10 and 15

(D) 15 and 10

16. If the maximum number of students has obtained 52 marks out of 80, then   **1**

(A) 52 is the mean of the data.

(B) 52 is the median of the data.

(C) 52 is the mode of the data.

(D) 52 is the range of the data.

17. समीकरण निकाय $2x + 1 = 0$ और $3y - 5 = 0$ का **1**

   (A) एक अद्वितीय हल है । (B) दो हल हैं ।

   (C) कोई भी हल नहीं है । (D) अपरिमित रूप से कई हल हैं ।

18. एक समकोण त्रिभुज ABC, जिसमें A पर समकोण है, में यदि $\sin B = \dfrac{1}{4}$, तो $\sec B$ का मान है **1**

   (A) 4 (B) $\dfrac{\sqrt{15}}{4}$

   (C) $\sqrt{15}$ (D) $\dfrac{4}{\sqrt{15}}$

   निर्देश : प्रश्न संख्या **19** तथा **20** प्रत्येक में एक **अभिकथन (A)** के पश्चात् एक **तर्क (R)** दिया है । निम्न में से सही विकल्प चुनिए :

   (A) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं और तर्क (R), अभिकथन (A) की पूर्ण व्याख्या करता है ।

   (B) अभिकथन (A) तथा तर्क (R) दोनों सत्य हैं, परन्तु तर्क (R), अभिकथन (A) की पूर्ण व्याख्या नहीं करता है ।

   (C) अभिकथन (A) सत्य है, परन्तु तर्क (R) असत्य है ।

   (D) अभिकथन (A) असत्य है, परन्तु तर्क (R) सत्य है ।

19. **अभिकथन (A) :** किन्हीं दो अभाज्य संख्याओं p और q के लिए, HCF होता है 1 और LCM होता है p + q । **1**

   तर्क (R) : किन्हीं दो प्राकृत संख्याओं के लिए HCF × LCM = संख्याओं का गुणनफल ।

20. पासा फेंकने के एक प्रयोग में **1**

   अभिकथन (A) : घटना E_1 : 3 से कम संख्या का प्राप्त होना तथा घटना E_2 : 3 से अधिक संख्या का प्राप्त होना पूरक घटनाएँ हैं ।

   तर्क (R) : यदि दो घटनाएँ E और F पूरक हैं, तो P(E) + P(F) = 1.

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17. The system of equations $2x + 1 = 0$ and $3y - 5 = 0$ has   **1**

(A) unique solution   (B) two solutions

(C) no solution   (D) infinite number of solutions

18. In a right triangle ABC, right-angled at A, if $\sin B = \frac{1}{4}$, then the value of

sec B is   **1**

(A) 4   (B) $\frac{\sqrt{15}}{4}$

(C) $\sqrt{15}$ (D) $\frac{4}{\sqrt{15}}$

Directions : In Question Numbers **19** and **20**, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option from the following :

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** For any two prime numbers p and q, their HCF is 1 and LCM is p + q.   **1**

     Reason (R) : For any two natural numbers, HCF × LCM = product of numbers.

20. In an experiment of throwing a die,   **1**

     Assertion (A) : Event E_1 : getting a number less than 3 and Event E_2 : getting a number greater than 3 are complementary events.

     Reason (R) : If two events E and F are complementary events, then $P(E) + P(F) = 1$.

खण्ड – ख

इस खण्ड में 5 अति लघु-उत्तरीय प्रश्न हैं । प्रत्येक प्रश्न के 2 अंक हैं ।

21. (a) निम्न समीकरण निकाय का बीजगणितीय विधि से हल कीजिए :   **2**

$$101x + 102y = 304$$

$$102x + 101y = 305$$

अथवा

(b) पूरक कोणों के एक युग्म में, बड़ा कोण छोटे कोण से 50° अधिक है । इस स्थिति को, दो चरों वाले रैखिक समीकरण निकाय में व्यक्त कीजिए और प्रत्येक कोण का माप ज्ञात कीजिए ।   **2**

22. (a) यदि $a \sec \theta + b \tan \theta = m$ तथा $b \sec \theta + a \tan \theta = n$ **2**

है, तो सिद्ध कीजिए कि $a^2 + n^2 = b^2 + m^2$

अथवा

(b) सर्वसमिका $\sin^2 A + \cos^2 A = 1$ का प्रयोग करके, सिद्ध कीजिए कि $\tan^2 A + 1 = \sec^2 A$

है । अतः $\tan A$ का मान ज्ञात कीजिए, जबकि $\sec A = \dfrac{5}{3}$ है, जहाँ A एक न्यूनकोण है ।   **2**

23. सिद्ध कीजिए कि बिंदु P, जो बिंदुओं A(7, 1) और B(3, 5) से समदूरस्थ है, का भुज, उसकी कोटि से 2 अधिक है ।   **2**

24. त्रिभुज ABC की भुजा BC पर एक बिंदु P इस प्रकार स्थित है कि $\angle APC = \angle BAC$ है । सिद्ध कीजिए कि $AC^2 = BC \cdot CP$.   **2**

25. एक थैले में लाल गेंदों की संख्या, काली गेंदों की संख्या से 3 अधिक है । इस थैले से यादृच्छया एक लाल गेंद निकलने की प्रायिकता यदि $\dfrac{12}{23}$ है, तो इस थैले में कुल गेंदों की संख्या ज्ञात कीजिए ।   **2**

• • •

SECTION – B

This section has 5 very short answer type questions of 2 marks each.

21. (a) Solve the following pair of equations algebraically :   **2**

$$101x + 102y = 304$$
$$102x + 101y = 305$$

OR

(b) In a pair of supplementary angles, the greater angle exceeds the smaller by 50°. Express the given situation as a system of linear equations in two variables and hence obtain the measure of each angle.   **2**

22. (a) If $a \sec \theta + b \tan \theta = m$ and $b \sec \theta + a \tan \theta = n$,   **2**

prove that $a^2 + n^2 = b^2 + m^2$

OR

(b) Use the identity : $\sin^2 A + \cos^2 A = 1$ to prove that $\tan^2 A + 1 = \sec^2 A$. Hence, find the value of $\tan A$, when $\sec A = \dfrac{5}{3}$, where A is an acute angle.   **2**

23. Prove that abscissa of a point P which is equidistant from points with coordinates A(7, 1) and B(3, 5) is 2 more than its ordinate.   **2**

24. P is a point on the side BC of ∆ABC such that $\angle APC = \angle BAC$. Prove that $AC^2 = BC \cdot CP$.   **2**

25. The number of red balls in a bag is three more than the number of black balls. If the probability of drawing a red ball at random from the given bag is $\dfrac{12}{23}$, find the total number of balls in the given bag.   **2**

●●●

इस खण्ड में 6 लघु-उत्तरीय प्रश्न हैं । प्रत्येक प्रश्न के 3 अंक हैं ।

26. (a) सिद्ध कीजिए कि $\sqrt{5}$ एक अपरिमेय संख्या है ।   3

अथवा

(b) माना p, q, r तीन विभिन्न अभाज्य संख्याएँ हैं ।
जाँच कीजिए कि p · q · r + q एक भाज्य संख्या है या नहीं ।
आगे, एक उदाहरण दीजिए जिसमें 3 विभिन्न अभाज्य संख्याओं p, q, r के लिए   3
(i)   p · q · r + 1 एक भाज्य संख्या है ।
(ii)   p · q · r + 1 एक अभाज्य संख्या है ।

27. बहुपद $p(x) = 3x^2 - 4x - 4$ के शून्यक ज्ञात कीजिए । अतः एक बहुपद ज्ञात कीजिए जिसका प्रत्येक शून्यक, $p(x)$ के शून्यकों से 2 अधिक हो ।   3

28. जाँच कीजिए कि क्या निम्न समीकरण निकाय   3
$$x + 3y = 6$$
$$3y - 2x = -12$$
संगत है या नहीं । यदि संगत है, तो इस समीकरण निकाय का हल ग्राफ द्वारा ज्ञात कीजिए ।

29. यदि बिंदु A(6, 1), B(p, 2), C(9, 4) और D(7, q) एक समांतर चतुर्भुज ABCD के शीर्ष हैं, तो p और q के मान ज्ञात कीजिए । यह भी जाँच कीजिए कि क्या यह समांतर चतुर्भुज एक आयत है या नहीं ।   3

30. (a) सिद्ध कीजिए कि : $\dfrac{\cos \theta - 2 \cos^3\theta}{\sin\theta - 2 \sin^3\theta} + \cot \theta = 0$.   3

अथवा

(b) दिया गया है कि $\sin \theta + \cos \theta = x$ है, सिद्ध कीजिए कि $\sin^4\theta + \cos^4\theta = \dfrac{2 - (x^2 - 1)^2}{2}$.   3

31. दी गई आकृति में, O केन्द्र वाले वृत्त पर TP और TQ दो स्पर्श-रेखाएँ हैं । यदि $\angle OPQ = 15°$ तथा $\angle PTQ = \theta$ है, तो $\sin 2\theta$ का मान ज्ञात कीजिए ।   3
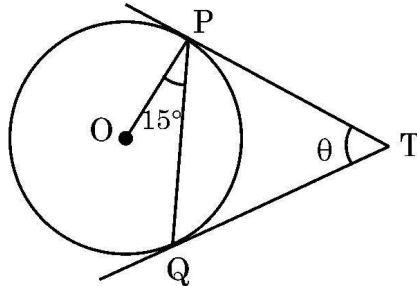
SECTION – C

This section has 6 short answer type questions of 3 marks each.

26. (a) Prove that $\sqrt{5}$ is an irrational number. **3**

    OR

    (b) Let p, q and r be three distinct prime numbers. **3**
    Check whether $p \cdot q \cdot r + q$ is a composite number or not.
    Further, give an example for 3 distinct primes p, q, r such that
    (i) $p \cdot q \cdot r + 1$ is a composite number.
    (ii) $p \cdot q \cdot r + 1$ is a prime number.

27. Find the zeroes of the polynomial $p(x) = 3x^2 - 4x - 4$. Hence, write a polynomial whose each of the zeroes is 2 more than zeroes of $p(x)$. **3**

28. Check whether the following pair of equations is consistent or not. If consistent, solve graphically **3**
 $x + 3y = 6$
    $3y - 2x = -12$

29. If the points A(6, 1), B(p, 2), C(9, 4) and D(7, q) are the vertices of a parallelogram ABCD, then find the values of p and q. Hence, check whether ABCD is a rectangle or not. **3**

30. (a) Prove that : $\dfrac{\cos\theta - 2\cos^3\theta}{\sin\theta - 2\sin^3\theta} + \cot\theta = 0.$ **3**

    OR

    (b) Given that $\sin\theta + \cos\theta = x$, prove that $\sin^4\theta + \cos^4\theta = \dfrac{2 - (x^2 - 1)^2}{2}.$ **3**

31. In the adjoining figure, TP and TQ are tangents drawn to a circle with centre O. If $\angle OPQ = 15°$ and $\angle PTQ = \theta$, then find the value of $\sin 2\theta$. **3**
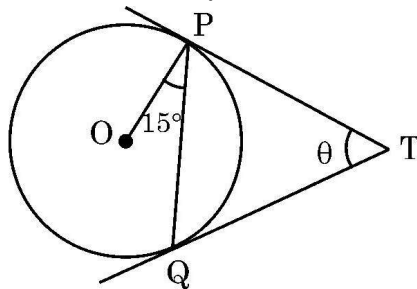
इस खण्ड में 4 दीर्घ-उत्तरीय प्रश्न हैं । प्रत्येक प्रश्न के 5 अंक हैं ।

32. (a) नीचे चित्र में, 65 m व्यास का एक वृत्ताकार पार्क दिखाया गया है, जिसमें AB एक व्यास है ।   **5**
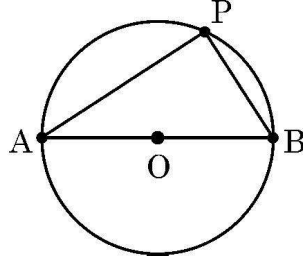
पार्क की सीमा पर एक बिंदु P पर एक प्रवेश द्वार इस प्रकार बनाया जाना है कि A से P की दूरी, B से P की दूरी से 35 m अधिक हो । बिन्दु P की दूरी बिन्दुओं A और B से ज्ञात कीजिए ।
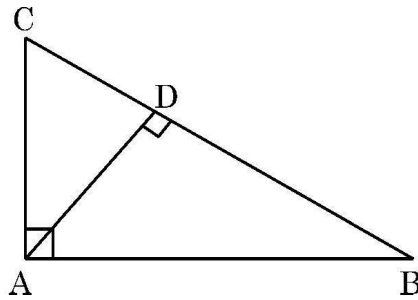
अथवा

(b) p का न्यूनतम मान ज्ञात कीजिए जिसके लिए द्विघात समीकरण $x^2 - 2(p + 1)x + p^2 = 0$ के मूल वास्तविक हों । अतः इस प्रकार प्राप्त द्विघात समीकरण के मूल भी ज्ञात कीजिए ।   **5**

33. (a) यदि एक रेखा किसी त्रिभुज की दो भुजाओं को एक ही अनुपात में विभाजित करे, तो वह तीसरी भुजा के समांतर होती है ।   **5**

ऊपर दिए गए कथन का विलोम लिखिए और उसे सिद्ध भी कीजिए ।

अथवा

(b) दी गई आकृति में, ΔCAB एक समकोण त्रिभुज है, जिसमें ∠A = 90° है और AD ⊥ BC है । सिद्ध कीजिए कि ΔADB ~ ΔCDA है । आगे, यदि BC = 10 cm और CD = 2 cm है, तो AD की लंबाई भी ज्ञात कीजिए ।   **5**

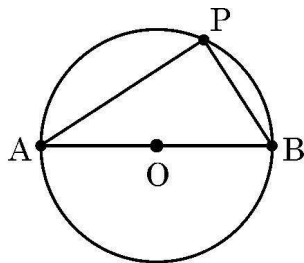34. 14 cm भुजा वाले एक ठोस घन के एक फलक से बड़ा-से-बड़ा एक शंकु खोद कर निकाला जाता है । बचे ठोस का आयतन और पृष्ठीय क्षेत्रफल ज्ञात कीजिए । $\left(\pi = \dfrac{22}{7} \text{ तथा } \sqrt{5} = 2.2 \text{ का प्रयोग करें ।} \right)$ **5**

• • •

SECTION – D

This section has 4 long answer questions of 5 marks each.

32. (a) There is a circular park of diameter 65 m as shown in the following figure, where AB is a diameter.   **5**

An entry gate is to be constructed at a point P on the boundary of the park such that distance of P from A is 35 m more than the distance of P from B. Find distance of point P from A and B respectively.

OR

(b) Find the smallest value of p for which the quadratic equation $x^2 - 2(p + 1)x + p^2 = 0$ has real roots. Hence, find the roots of the equation so obtained.   **5**

33. (a) If a line drawn parallel to one side of triangle intersecting the other two sides in distinct points divides the two sides in the same ratio, then it is parallel to third side.   **5**

State and prove the converse of the above statement.

OR

(b) In the adjoining figure, ΔCAB is a right triangle, right angled at A and AD ⊥ BC. Prove that ΔADB ~ ΔCDA. Further, if BC = 10 cm and CD = 2 cm, find the length of AD.   **5**
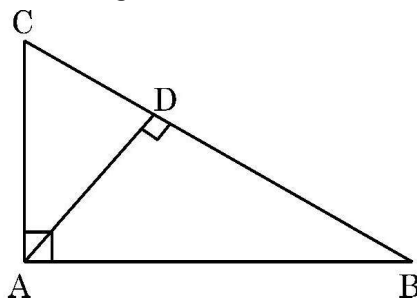
34. From one face of a solid cube of side 14 cm, the largest possible cone is carved out. Find the volume and surface area of the remaining solid.

$\left(\text{Use } \pi = \dfrac{22}{7}, \sqrt{5} = 2.2 \right)$ **5**

35. निम्न बंटन में, एक विशेष विषय में 230 विद्यार्थियों के प्राप्तांक दर्शाए गए हैं । यदि माध्यक अंक 46 हैं, तो x और y के मान ज्ञात कीजिए ।
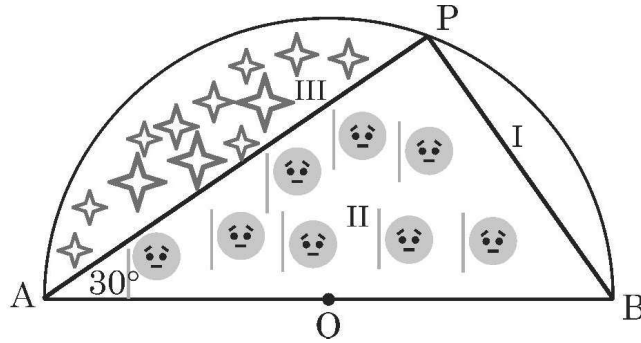
5

प्राप्तांक	विद्यार्थियों की संख्या
10 – 20	12
20 – 30	30
30 – 40	x
40 – 50	65
50 – 60	y
60 – 70	25
70 – 80	18

खण्ड – ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं । प्रत्येक प्रश्न के 4 अंक हैं ।

36. अनुराग ने एक फार्म-हाऊस खरीदा जो 70 m व्यास वाले अर्धवृत्त के रूप में है । वह इसे अर्धवृत्त पर एक बिंदु P लेकर इस प्रकार तीन भागों में विभाजित करता है कि $\angle PAB = 30°$ है, जैसा कि नीचे दी गई आकृति में दिखाया गया है, O अर्धवृत्त का केन्द्र है ।

भाग I में उन्होंने आम के पेड़ के पौधे, भाग II में टमाटर उगाए और भाग III में, उन्होंने संतरे उगाए ।

उपरोक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) $\angle POA$ का माप क्या है ? 1

(ii) भूमि के पूरे टुकड़े की बाड़ लगाने के लिए आवश्यक तार की लंबाई ज्ञात कीजिए । 1

(iii) (a) जिस क्षेत्र में आम के पेड़ के पौधे लगाए गए हैं, उसका क्षेत्रफल ज्ञात कीजिए । 2

अथवा

(iii) (b) क्षेत्र III के चारों तरफ बाड़ लगाने के लिए आवश्यक तार की लंबाई ज्ञात कीजिए । 2

● ● ●

35. Following distribution shows the marks of 230 students in a particular subject. If the median marks are 46, then find the values of x and y. **5**

Marks	Number of Students
10 – 20	12
20 – 30	30
30 – 40	x
40 – 50	65
50 – 60	y
60 – 70	25
70 – 80	18

SECTION – E

This section has 3 case study based questions of 4 marks each.

36. Anurag purchased a farmhouse which is in the form of a semicircle of diameter 70 m. He divides it into three parts by taking a point P on the semicircle in such a way that $\angle PAB = 30°$ as shown in the following figure, where O is the centre of semicircle.
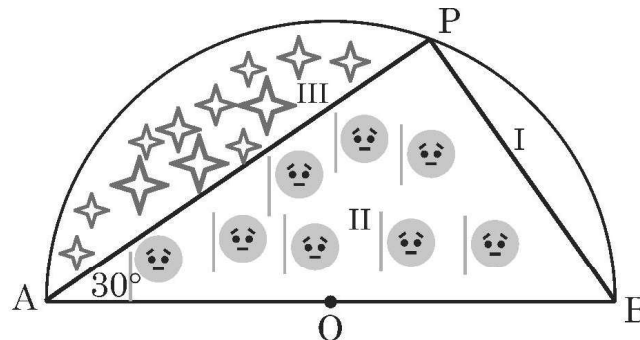
In part I, he planted saplings of Mango tree, in part II, he grew tomatoes and in part III, he grew oranges. Based on given information, answer the following questions.

(i)   What is the measure of $\angle POA$?   **1**

(ii) Find the length of wire needed to fence entire piece of land.   **1**

(iii) (a) Find the area of region in which saplings of Mango tree are planted.   **2**

OR

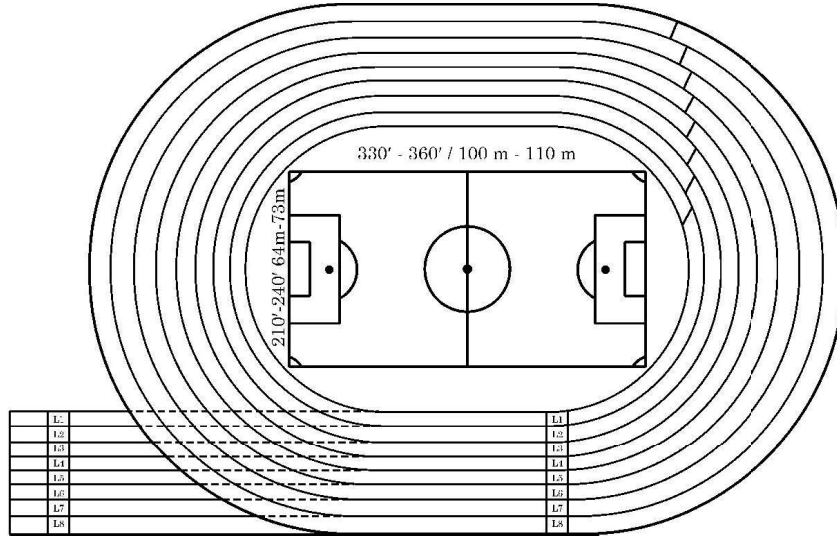(iii) (b) Find the length of wire needed to fence the region III.   **2**

37. वार्षिक खेल दिवस आयोजित करने के लिए, एक स्कूल ने ट्रैक क्षेत्र के अंदर एक एकीकृत फुटबॉल मैदान के साथ आठ लेन का रनिंग ट्रैक तैयार किया जैसा कि नीचे चित्र में दिखाया गया है :

ट्रैक की सबसे भीतरी लेन की लंबाई 400 m है और प्रत्येक अगली लेन पिछली लेन से 7.6 m लंबी है ।

उपरोक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर, समांतर श्रेढ़ी की अवधारणा के प्रयोग से दीजिए ।

(i)   छठी लेन की लंबाई कितनी है ?   1

(ii) 8वीं लेन की लम्बाई, चौथी लेन की लंबाई से कितनी अधिक है ?   1

(iii) (a) एक दौड़ के लिए अभ्यास करते हुए, एक छात्र ने पहली छह लेन में एक-एक चक्कर लगाया । छात्र द्वारा तय की गई कुल दूरी ज्ञात कीजिए ।   2

अथवा

(iii) (b) एक छात्र ने लेन चौथी से लेन 8वीं तक प्रत्येक में एक-एक चक्कर लगाया । छात्र द्वारा तय की गई कुल दूरी ज्ञात कीजिए ।   2

38. गुजरात में स्थित स्टैच्यू ऑफ यूनिटी दुनिया की सबसे बड़ी प्रतिमा है जो 58 m ऊँचे चबूतरे पर खड़ी है । परियोजना के रूप में, एक छात्र ने एक इनक्लिनोमीटर का निर्माण किया और इसका उपयोग करके स्टैच्यू ऑफ यूनिटी की ऊँचाई ज्ञात करना चाहता है ।

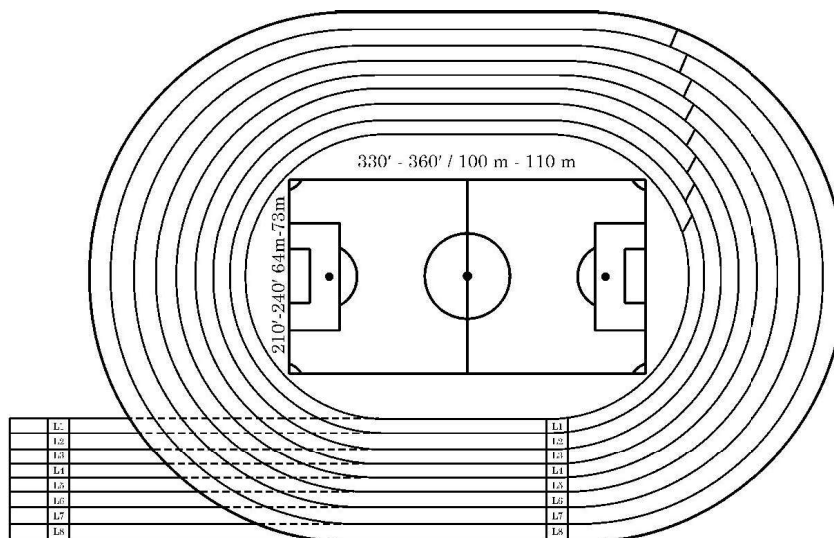उसने दो स्थानों से निम्नलिखित टिप्पणियाँ नोट की :

स्थिति – I :

स्थान A से, जो आधार से $80\sqrt{3}$ m की दूरी पर है, मूर्ति के शीर्ष का उन्नयन कोण 60° पाया जाता है ।

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37. In order to organise, Annual Sports Day, a school prepared an eight lane running track with an integrated football field inside the track area as shown below :

The length of innermost lane of the track is 400 m and each subsequent lane is 7.6 m longer than the preceding lane.

Based on given information, answer the following questions, using concept of Arithmetic Progression.

(i)   What is the length of the 6th lane ?   **1**

(ii)   How long is the 8th lane than that of 4th lane ?   **1**

(iii) (a) While practicing for a race, a student took one round each in first six lanes. Find the total distance covered by the student.   **2**

OR

(iii) (b) A student took one round each in lane 4 to lane 8. Find the total distance covered by the student.   **2**

38. The Statue of Unity situated in Gujarat is the world's largest Statue which stands over a 58 m high base. As part of the project, a student constructed an inclinometer and wishes to find the height of Statue of Unity using it.

He noted following observations from two places :

Situation – I :

The angle of elevation of the top of Statue from Place A which is $80\sqrt{3}$ m away from the base of the Statue is found to be 60°.

स्थिति – II :

स्थान B से, जो आधार से 40 m की दूरी पर है, मूर्ति के शीर्ष का उन्नयन कोण 30° पाया जाता है और मूर्ति की कुल ऊँचाई 240 m, जिसमें B की भूमि से ऊँचाई भी सम्मिलित है, पायी जाती है ।

उपरोक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i)   स्थिति – I को आरेख (चित्र) की सहायता से निरूपित कीजिए ।   1

(ii) स्थिति – II को आरेख (चित्र) की सहायता से निरूपित कीजिए ।   1

(iii) (a) आधार को छोड़कर मूर्ति की ऊँचाई की गणना करिए और स्थिति – I की सहायता से आधार सहित ऊँचाई भी ज्ञात करिए ।   2

अथवा

(iii) (b) मूर्ति से बिंदु B (स्थिति – II) की क्षैतिज दूरी और $\tan \alpha$ का मान ज्ञात करिए, जहाँ α बिंदु B से मूर्ति के आधार के शीर्ष का उन्नयन कोण है ।   2

———————

● ● ●

Situation – II :

The angle of elevation of the top of Statue from a Place B which is 40 m above the ground is found to be 30° and entire height of the Statue including the base is found to be 240 m.

Based on given information, answer the following questions :

(i)   Represent the Situation – I with the help of a diagram.   **1**

(ii) Represent the Situation – II with the help of a diagram.   **1**

(iii) (a) Calculate the height of Statue excluding the base and also find
           the height including the base with the help of Situation – I.   **2**

<div align="center">

OR

</div>

(iii) (b) Find the horizontal distance of point B (Situation – II) from the
           Statue and the value of tan α, where α is the angle of elevation
           of top of base of the Statue from point B.   **2**

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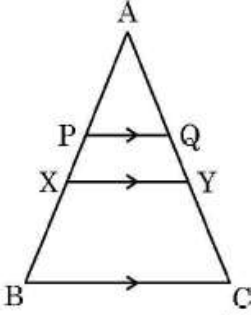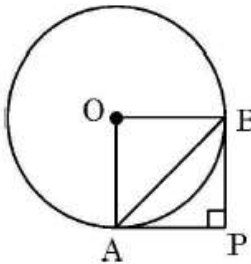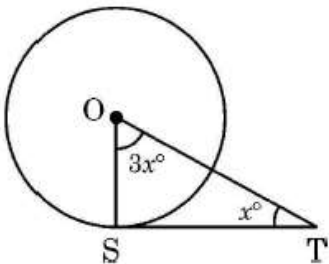|	**Marking Scheme** **Strictly Confidential** **(For Internal and Restricted use only)** **Secondary School Examination, 2025** **MATHEMATICS (Standard) (Q.P. CODE 30/6/1)**

General Instructions: -

1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. It's leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-X, while evaluating the competency-based questions, please try to understand given answer and even if reply is not from Marking Scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4.	The Marking scheme carries only suggested value points for the answers. These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written on the left-hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded on the left-hand margin and encircled. This may also be followed strictly.

9.	If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11.	A full scale of marks _____80_____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12.	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totalling of marks awarded to an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totalling on the title page. ● Wrong totalling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
18.	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

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Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **This section consists of 20 multiple choice questions of 1 mark each.**	
1.	$\sqrt{0.4}$ is a/an (A) natural number (B) integer (C) rational number (D) irrational number	
Sol.	(D) irrational number	1
2.	Which of the following cannot be the unit digit of 8^n, where n is a natural number ? (A) 4 (B) 2 (C) 0 (D) 6	
Sol.	(C) 0	1
3.	Which of the following quadratic equations has real and equal roots ? (A) $(x + 1)^2 = 2x + 1$ (B) $x^2 + x = 0$ (C) $x^2 - 4 = 0$ (D) $x^2 + x + 1 = 0$	
Sol.	(A) $(x + 1)^2 = 2x + 1$	1
4.	If the zeroes of the polynomial $ax^2 + bx + \dfrac{2a}{b}$ are reciprocal of each other, then the value of b is (A) 2 (B) $\dfrac{1}{2}$ (C) -2 (D) $-\dfrac{1}{2}$	
Sol.	(A) 2	1
5.	The distance of the point A(-3, -4) from x-axis is (A) 3 (B) 4 (C) 5 (D) 7	
Sol.	(B) 4	1
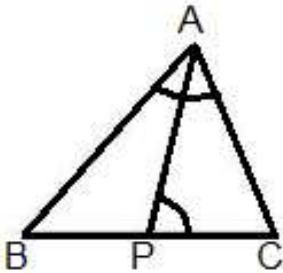
6.	In the adjoining figure, PQ ∥ XY ∥ BC, AP = 2 cm, PX = 1.5 cm and BX = 4 cm. If QY = 0.75 cm, then AQ + CY = ![figure of triangle with PQ, XY parallel to BC] (A) 6 cm (B) 4.5 cm (C) 3 cm (D) 5.25 cm	
Sol.	(C) 3 cm	**1**
7.	Given ΔABC ~ ΔPQR, ∠A = 30° and ∠Q = 90°. The value of (∠R + ∠B) is (A) 90° (B) 120° (C) 150° (D) 180°	
Sol.	(C) 150°	**1**
8.	Two coins are tossed simultaneously. The probability of getting atleast one head is (A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{3}{4}$ (D) 1	
Sol.	(C) $\dfrac{3}{4}$	**1**
9.	In the adjoining figure, PA and PB are tangents to a circle with centre O such that ∠P = 90°. If AB = $3\sqrt{2}$ cm, then the diameter of the circle is ![figure of circle with tangents PA, PB] (A) $3\sqrt{2}$ cm (B) $6\sqrt{2}$ cm (C) 3 cm (D) 6 cm	
Sol.	(D) 6 cm	**1**

10.	For a circle with centre O and radius 5 cm, which of the following statements is true ? **P** : Distance between every pair of parallel tangents is 5 cm. **Q** : Distance between every pair of parallel tangents is 10 cm. **R** : Distance between every pair of parallel tangents must be between 5 cm and 10 cm. **S** : There does not exist a point outside the circle from where length of tangent is 5 cm. (A) P (B) Q (C) R (D) S	
Sol.	(B) Q	**1**
11.	In the adjoining figure, TS is a tangent to a circle with centre O. The value of $2x°$ is (A) 22.5 (B) 45 (C) 67.5 (D) 90	
Sol.	(B) 45	**1**
12.	If $x\left(\dfrac{2 \tan 30°}{1 + \tan^2 30°}\right) = y\left(\dfrac{2 \tan 30°}{1 - \tan^2 30°}\right)$, then $x : y =$ (A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1	
Sol.	(C) 2:1	**1**
13.	A peacock sitting on the top of a tree of height 10 m observes a snake moving on the ground. If the snake is $10\sqrt{3}$ m away from the base of the tree, then angle of depression of the snake from the eye of the peacock is (A) 30° (B) 45° (C) 60° (D) 90°	
Sol.	(A) 30°	**1**

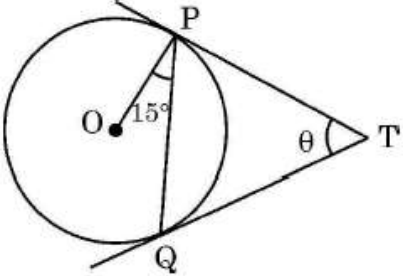| 14.	If a cone of greatest possible volume is hollowed out from a solid wooden cylinder, then the ratio of the volume of remaining wood to the volume of cone hollowed out is (A) 1 : 1 (B) 1 : 3 (C) 2 : 1 (D) 3 : 1	
Sol.	(C) 2:1	**1**
15.	If the mode of some observations is 10 and sum of mean and median is 25, then the mean and median respectively are (A) 12 and 13 (B) 13 and 12 (C) 10 and 15 (D) 15 and 10	
Sol.	(B) 13 and 12	**1**
16.	If the maximum number of students has obtained 52 marks out of 80, then (A) 52 is the mean of the data. (B) 52 is the median of the data. (C) 52 is the mode of the data. (D) 52 is the range of the data.	
Sol.	(C) 52 is the mode of the data.	**1**
17.	The system of equations $2x + 1 = 0$ and $3y - 5 = 0$ has (A) unique solution (B) two solutions (C) no solution (D) infinite number of solutions	
Sol.	(A) unique solution	**1**
18.	In a right triangle ABC, right-angled at A, if $\sin B = \dfrac{1}{4}$, then the value of sec B is (A) 4 (B) $\dfrac{\sqrt{15}}{4}$ (C) $\sqrt{15}$ (D) $\dfrac{4}{\sqrt{15}}$	
Sol.	(D) $\dfrac{4}{\sqrt{15}}$	**1**

	Directions : In Question Numbers **19** and **20**, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option from the following : (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	**Assertion (A) :** For any two prime numbers p and q, their HCF is 1 and LCM is p + q. **Reason (R)** : For any two natural numbers, HCF × LCM = product of numbers.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true.	**1**
20.	In an experiment of throwing a die, **Assertion (A) :** Event E_1 : getting a number less than 3 and Event E_2 : getting a number greater than 3 are complementary events. **Reason (R)** : If two events E and F are complementary events, then P(E) + P(F) = 1.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true.	**1**
	SECTION B **This section has 5 very short answer type questions of 2 marks each.**	
21. (a)	Solve the following pair of equations algebraically : $101x + 102y = 304$ $102x + 101y = 305$	
Sol.	Adding equations we get $x + y = 3$ Subtracting equations we get $-x + y = -1$ Solving to get $x = 2$ and $y = 1$	½ ½ ½ + ½
	OR	

21. (b)	In a pair of supplementary angles, the greater angle exceeds the smaller by 50°. Express the given situation as a system of linear equations in two variables and hence obtain the measure of each angle.	
Sol.	Let smaller angle be x and greater angle be y ATQ, $x + y = 180$ Also $y = x + 50$ Solving we get $x = 65°$ and y $= 115°$	½ ½ ½ + ½
22. (a)	If a sec θ + b tan θ = m and b sec θ + a tan θ = n, prove that $a^2 + n^2 = b^2 + m^2$	
Sol.	$m^2 = a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta \tan\theta$ $n^2 = b^2 \sec^2\theta + a^2 \tan^2\theta + 2ab \sec\theta \tan\theta$ $m^2 - n^2 = a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta)$ $\Rightarrow m^2 - n^2 = a^2 - b^2$ or $a^2 + n^2 = m^2 + b^2$	½ ½ ½ ½
	OR	
22. (b)	Use the identity : $\sin^2 A + \cos^2 A = 1$ to prove that $\tan^2 A + 1 = \sec^2 A$. Hence, find the value of tan A, when sec A $= \dfrac{5}{3}$, where A is an acute angle.	
Sol.	$\sin^2 A + \cos^2 A = 1$ Dividing both sides by $\cos^2 A$, we get $\dfrac{\sin^2 A}{\cos^2 A} + \dfrac{\cos^2 A}{\cos^2 A} = \dfrac{1}{\cos^2 A}$ $\tan^2 A + 1 = \sec^2 A$ $\tan^2 A + 1 = \left(\dfrac{5}{3}\right)^2$ $\tan A = \dfrac{4}{3}$	 ½ ½ ½ ½
23.	Prove that abscissa of a point P which is equidistant from points with coordinates A(7, 1) and B(3, 5) is 2 more than its ordinate.	
Sol.	Let P (x, y) be equidistant from A(7, 1) and B(3, 5) PA = PB $\Longrightarrow$ PA^2 = PB^2 $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$ $x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$	 ½ ½ ½

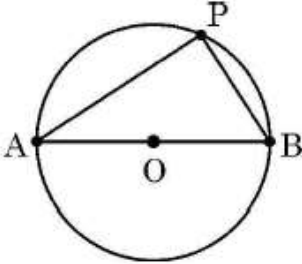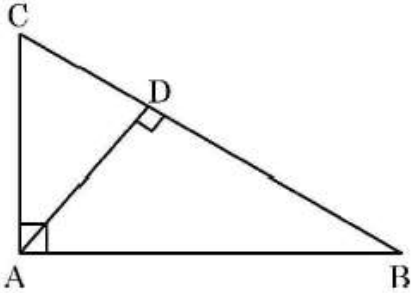|			
	$x = 2 + y$ Thus, abscissa of the point P is 2 more than its ordinate.		½
24.	P is a point on the side BC of ΔABC such that ∠APC = ∠BAC. Prove that $AC^2 = BC \cdot CP$.		
Sol.	 ∠ APC = ∠ BAC ∠ ACP = ∠ ACB ∴ Δ APC ~ Δ BAC $AC^2 = BC.CP$	Correct figure	½ 1 ½
25.	The number of red balls in a bag is three more than the number of black balls. If the probability of drawing a red ball at random from the given bag is $\frac{12}{23}$, find the total number of balls in the given bag.		
Sol.	Let number of black balls = x then number of red balls = $x + 3$ ∴ total number of balls = $2x + 3$ ATQ, $\frac{x+3}{2x+3} = \frac{12}{23}$ $x = 33$ Total number of balls = 69		½ ½ ½ ½
SECTION C **This section has 6 short answer type questions of 3 marks each.**		
26. **(a)**	Prove that $\sqrt{5}$ is an irrational number.		
Sol.	Let $\sqrt{5}$ be a rational number. ∴ $\sqrt{5} = \frac{p}{q}$, where q ≠ 0 and let p & q be the coprimes. $\Longrightarrow 5q^2 = p^2$ $\Longrightarrow p^2$ is divisible by 5. $\Longrightarrow p$ is divisible by 5. ----- ① Let p = 5a, where 'a' is some integer ∴ $25a^2 = 5q^2$ $\Longrightarrow q^2 = 5a^2$		½ 1

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	$\Rightarrow q^2$ is divisible by 5. $\Rightarrow q$ is divisible by 5. ----- ② $\therefore$ 5 divides both p & q.		**1**
	① and ② leads to contradiction as p and q are coprimes. Hence, $\sqrt{5}$ is an irrational number.		**½**
	OR		
26. (b)	Let p, q and r be three distinct prime numbers. Check whether $p \cdot q \cdot r + q$ is a composite number or not. Further, give an example for 3 distinct primes p, q, r such that (i) $p \cdot q \cdot r + 1$ is a composite number. (ii) $p \cdot q \cdot r + 1$ is a prime number.		

Sol.	p. q. r + q = q(pr + 1)
	 Thus, the given number has more than 2 factors.
	 Hence it is composite.
	 (i) Taking p = 3, q = 5 and r = 7
	    pqr + 1 = 3.5.7 + 1 = 106 is a composite number
	                 or any other correct example
	
	 (ii) Taking p = 2, q = 3 and r = 5
	    pqr + 1 = 2.3.5 + 1 = 31 is a prime number
	                 or any other correct example
			½
			 ½
			
			 1
			
			 1

27.	Find the zeroes of the polynomial $p(x) = 3x^2 - 4x - 4$. Hence, write a polynomial whose each of the zeroes is 2 more than zeroes of $p(x)$.		
Sol.	$p(x) = 3x^2 - 4x - 4$ Zeroes are $-\dfrac{2}{3}$ and 2 New zeroes are $\dfrac{4}{3}$ and 4 Sum of new zeroes $= \dfrac{4}{3} + 4 = \dfrac{16}{3}$ Product of new zeroes $= \dfrac{4}{3} \times 4 = \dfrac{16}{3}$ Required polynomial is $x^2 - \dfrac{16x}{3} + \dfrac{16}{3}$ or $3x^2 - 16x + 16$		**1** **½** **½** **½** **½**

28.	Check whether the following pair of equations is consistent or not. If consistent, solve graphically
	     $x + 3y = 6$
	     $3y - 2x = -12$		

Sol.	$x + 3y = 6$ $-2x + 3y = -12$ $\dfrac{a_1}{a_2} = \dfrac{1}{-2}$; $\dfrac{b_1}{b_2} = \dfrac{3}{3} = 1$		**½**

	$\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ Hence the pair of equations is consistent. Solution is (6, 0) or $x = 6$ and $y = 0$	½ Correct graph 1½ ½
29.	If the points A(6, 1), B(p, 2), C(9, 4) and D(7, q) are the vertices of a parallelogram ABCD, then find the values of p and q. Hence, check whether ABCD is a rectangle or not.	
Sol.	Diagonals of a parallelogram bisect each other. $\therefore$ Co-ordinates of mid point of diagonal AC = Co-ordinates of mid-point of diagonal BD. $$\left(\dfrac{6+9}{2}, \dfrac{1+4}{2}\right) = \left(\dfrac{p+7}{2}, \dfrac{2+q}{2}\right)$$ $$\Rightarrow \dfrac{p+7}{2} = \dfrac{15}{2} \text{ and } \dfrac{2+q}{2} = \dfrac{5}{2}$$ $$\therefore p = 8 \text{ and } q = 3$$ Diagonal AC $= \sqrt{3^2 + 3^2} = 3\sqrt{2}$ Diagonal BD $= \sqrt{(-1)^2 + 1^2} = \sqrt{2}$ AC $\neq$ BD $\therefore$ ABCD is not a rectangle	1 ½ ½ ½ ½
30. (a)	Prove that : $\dfrac{\cos\theta - 2\cos^3\theta}{\sin\theta - 2\sin^3\theta} + \cot\theta = 0$.	
Sol.	LHS $= \dfrac{\cos\theta - 2\cos^3\theta}{\sin\theta - 2\sin^3\theta} + \cot\theta$ $= \dfrac{\cos\theta(1 - 2\cos^2\theta)}{\sin\theta(1 - 2\sin^2\theta)} + \cot\theta$ $= \dfrac{\cos\theta}{\sin\theta}\left[\dfrac{\sin^2\theta + \cos^2\theta - 2\cos^2\theta}{\sin^2\theta + \cos^2\theta - 2\sin^2\theta}\right] + \cot\theta$ $= \dfrac{\cot\theta(\sin^2\theta - \cos^2\theta)}{(\cos^2\theta - \sin^2\theta)} + \cot\theta$ $= -\cot\theta + \cot\theta$ $= 0 = $ RHS	 ½ 1 1 ½

	OR	
30. (b)	Given that $\sin\theta + \cos\theta = x$, prove that $\sin^4\theta + \cos^4\theta = \dfrac{2 - (x^2 - 1)^2}{2}$.	
Sol.	Given: $\sin\theta + \cos\theta = x$ Squaring both sides $\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta = x^2$ $2\sin\theta\cos\theta = x^2 - 1$	**1**
	RHS $= \dfrac{2 - (2\sin\theta\cos\theta)^2}{2}$	½
	$\quad = \dfrac{2 - 4\sin^2\theta\cos^2\theta}{2}$	
	$\quad = 1 - 2\sin^2\theta\cos^2\theta$	½
	$\quad = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$	½
	$\quad = (\sin^4\theta + \cos^4\theta) = $ LHS	½
31.	In the adjoining figure, TP and TQ are tangents drawn to a circle with centre O. If $\angle OPQ = 15°$ and $\angle PTQ = \theta$, then find the value of $\sin 2\theta$. 	
Sol.	$\angle$ QPT $= 75°$	½
	$\angle$ PQT $= 75°$	½
	$\theta = 30°$	**1**
	$\sin 2\theta = \sin 2(30°)$	½
	$\quad = \sin 60° = \dfrac{\sqrt{3}}{2}$	½

SECTION D

This section has 4 long answer questions of 5 marks each.

32. (a)	There is a circular park of diameter 65 m as shown in the following figure, where AB is a diameter. An entry gate is to be constructed at a point P on the boundary of the park such that distance of P from A is 35 m more than the distance of P from B. Find distance of point P from A and B respectively.	
Sol.	Let distance of gate at P from point B is x m Then distance of gate at P from point A is $(35 + x)$ m In right Δ APB $(x + 35)^2 + x^2 = (65)^2$ $x^2 + 35x - 1500 = 0$ $(x + 60)(x - 25) = 0$ $x = 25$ Hence, $x + 35 = 60$ Distance of P from A = 60 m Distance of P from B = 25 m	½ 1 2 ½ ½ ½
	OR	
32. (b)	Find the smallest value of p for which the quadratic equation $x^2 - 2(p + 1)x + p^2 = 0$ has real roots. Hence, find the roots of the equation so obtained.	
Sol.	For real roots, D ≥ 0 $[- 2(p + 1)]^2 - 4p^2 \geq 0$ $\Rightarrow p \geq -\dfrac{1}{2}$ ∴ smallest value of $p = -\dfrac{1}{2}$ At $p = -\dfrac{1}{2}$ given equation becomes $x^2 - 2\left(\dfrac{-1}{2} + 1\right)x + \left(\dfrac{-1}{2}\right)^2 = 0$ $x^2 - x + \dfrac{1}{4} = 0$ or $4x^2 - 4x + 1 = 0$ $(2x - 1)(2x - 1) = 0$ ∴ roots are $\dfrac{1}{2}, \dfrac{1}{2}$	½ ½ 1 ½ ½ 1 ½ ½

33. (a)	If a line drawn parallel to one side of triangle intersecting the other two sides in distinct points divides the two sides in the same ratio, then it is parallel to third side. State and prove the converse of the above statement.	
Sol.	Correct Statement of BPT Correct figure, Given, To Prove, Construction Correct Proof of BPT **NOTE*** Given statement in English version is not a correct statement. Full marks may be awarded to any attempt in English medium.	**1** **2** **2**
	OR	
33. (b)	In the adjoining figure, ΔCAB is a right triangle, right angled at A and AD ⊥ BC. Prove that ΔADB ~ ΔCDA. Further, if BC = 10 cm and CD = 2 cm, find the length of AD. 	
Sol.	Δ ABC ~ Δ DAC ----- ① Similarly, Δ ABC ~ Δ DBA ----- ② From equations ① and ② Δ DAC ~ Δ DBA or Δ ADB ~ Δ CDA $\dfrac{AD}{CD} = \dfrac{BD}{AD}$ $AD^2 = BD \times CD$ $= 8 \times 2$ $\therefore AD = 4$ cm.	**1** **½** **1** **½** **½** **1** **½**
34.	From one face of a solid cube of side 14 cm, the largest possible cone is carved out. Find the volume and surface area of the remaining solid. $\left(\text{Use } \pi = \dfrac{22}{7}, \sqrt{5} = 2.2\right)$	
Sol.	Diameter of cone = 14 cm Radius = 7 cm Height of cone = 14 cm Slant height $l = \sqrt{14^2 + 7^2} = 7\sqrt{5} = 15.4$ cm Volume of remaining solid = Volume of cube – Volume of cone $\qquad = (14)^3 - \dfrac{1}{3} \times \dfrac{22}{7} \times (7)^2 \times 14$ $\qquad = \dfrac{6076}{3}$ cm³	 **½** **½** **1** **1** **½**

	Surface area of remaining solid = Surface area of cube – Area of circle + Curved surface area of cone	
	$$= 6 \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 15.4$$ $$= 1360.8 \text{ cm}^2$$	**1** ½
35.	Following distribution shows the marks of 230 students in a particular subject. If the median marks are 46, then find the values of x and y.	

Marks	Number of Students
10 – 20	12
20 – 30	30
30 – 40	x
40 – 50	65
50 – 60	y
60 – 70	25
70 – 80	18

Sol.

Marks	Number of Students	Cf
10 – 20	12	12
20 – 30	30	42
30 – 40	x	42 + x
40 – 50	65	107 + x
50 – 60	y	107 + x + y
60 – 70	25	132 + x + y
70 – 80	18	150 + x + y
	230	

Correct table **1**

$150 + x + y = 230$

x + y = 80 **1**

Median is 46

∴ Median class is 40 – 50 **½**

$$46 = 40 + \left[\frac{\frac{230}{2} - (42 + x)}{65}\right] \times 10$$ **1**

On solving, we get $x = 34$ **1**

and y = 46 **½**

	SECTION E	
	This section has 3 case study-based questions of 4 marks each.	
36.	Anurag purchased a farmhouse which is in the form of a semicircle of diameter 70 m. He divides it into three parts by taking a point P on the semicircle in such a way that ∠PAB = 30° as shown in the following figure, where O is the centre of semicircle. In part I, he planted saplings of Mango tree, in part II, he grew tomatoes and in part III, he grew oranges. Based on given information, answer the following questions. (i) What is the measure of ∠POA ? (ii) Find the length of wire needed to fence entire piece of land. (iii) (a) Find the area of region in which saplings of Mango tree are planted. <center>**OR**</center> (iii) (b) Find the length of wire needed to fence the region III.	
Sol.	(i) ∠ POA = 120°	**1**
	(ii) Length of wire needed to fence entire piece of land $= \dfrac{22}{7} \times 35 + 70 = 180$ m	**1**
	(iii) Required area $= \dfrac{60}{360} \times \dfrac{22}{7} \times (35)^2 - \dfrac{\sqrt{3}}{4} \times (35)^2$	**1**
	$\qquad = \left(\dfrac{1925}{3} - \dfrac{1225\sqrt{3}}{4} \right)$ m^2 or 111.89 m^2 (approx.)	**1**
	<center>**OR**</center>	
	(iii) In Δ APB, $\dfrac{AP}{AB} = \cos 30°$	
	$\qquad AP = 35\sqrt{3}$ m	**1**
	$\quad$ Required length of wire $= \dfrac{120}{360} \times 2 \times \dfrac{22}{7} \times 35 + 35\sqrt{3}$	**½**
	$\qquad\qquad = \left(\dfrac{220}{3} + 35\sqrt{3} \right)$ m or 133.8 m (approx.)	**½**
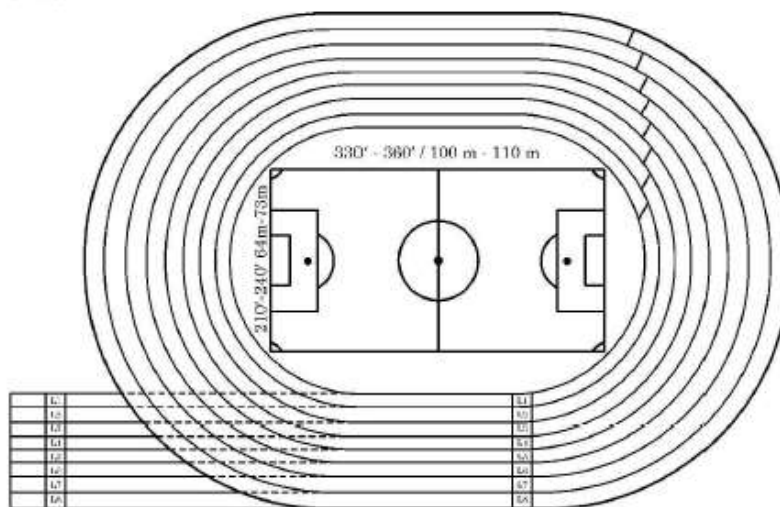
37.	In order to organise, Annual Sports Day, a school prepared an eight lane running track with an integrated football field inside the track area as shown below :	
	330' - 360' / 100 m - 110 m	
	The length of innermost lane of the track is 400 m and each subsequent lane is 7.6 m longer than the preceding lane.	
	Based on given information, answer the following questions, using concept of Arithmetic Progression.	
	(i) What is the length of the 6th lane ?	
	(ii) How long is the 8th lane than that of 4th lane ?	
	(iii) (a) While practicing for a race, a student took one round each in first six lanes. Find the total distance covered by the student.	
	OR	
	(iii) (b) A student took one round each in lane 4 to lane 8. Find the total distance covered by the student.	
Sol.	Here AP is 400, 407.6, 415.2, …	
	(i) $a_6 = 400 + 5 (7.6) = 438$ m	**1**
	(ii) $a_8 - a_4 = 30.4$ m	**1**
	(iii) $S_6 = \dfrac{6}{2}$ (2 × 400 + 5 × 7.6)	**1**
	$= 2514$ m	**1**
	OR	
	(iii) Total distance covered $= S_8 - S_3$	
	$= \dfrac{8}{2}$ (2 × 400 + 7 × 7.6) $- \dfrac{3}{2}$ (2 × 400 + 2 × 7.6)	**1**
	$= 2190$ m	**1**

38. The Statue of Unity situated in Gujarat is the world's largest Statue which stands over a 58 m high base. As part of the project, a student constructed an inclinometer and wishes to find the height of Statue of Unity using it.

He noted following observations from two places :

Situation – I :

The angle of elevation of the top of Statue from Place A which is $80\sqrt{3}$ m away from the base of the Statue is found to be 60°.

Situation – II :

The angle of elevation of the top of Statue from a Place B which is 40 m above the ground is found to be 30° and entire height of the Statue including the base is found to be 240 m.
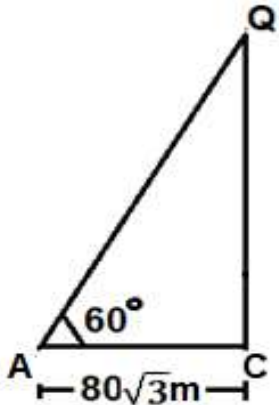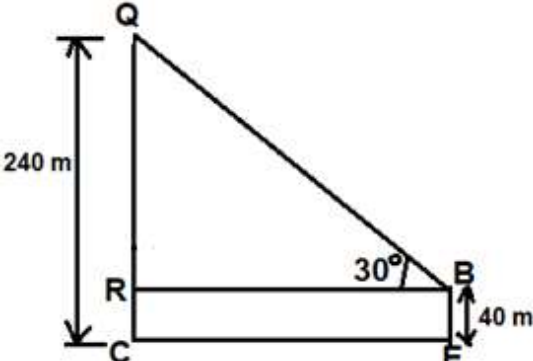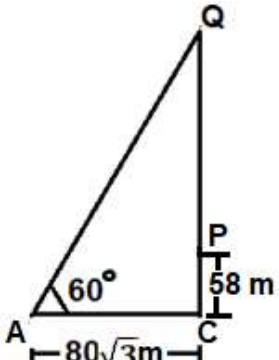
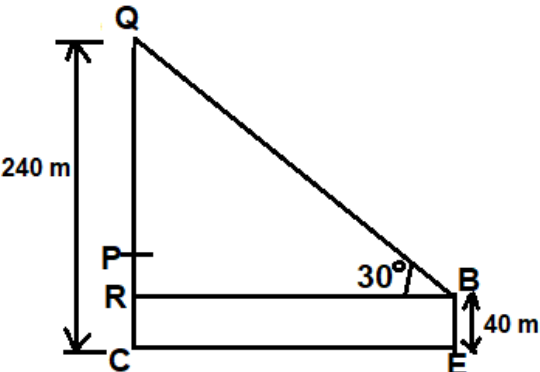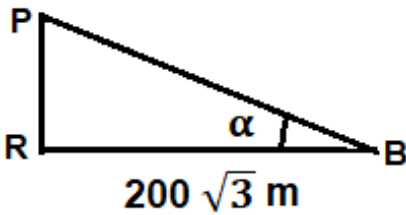Based on given information, answer the following questions :

(i)   Represent the Situation – I with the help of a diagram.

(ii) Represent the Situation – II with the help of a diagram.

(iii) (a)   Calculate the height of Statue excluding the base and also find the height including the base with the help of Situation – I.

OR

(iii) (b)   Find the horizontal distance of point B (Situation – II) from the Statue and the value of tan α, where α is the angle of elevation of top of base of the Statue from point B.

Sol.	(i) 	Correct figure	**1**
	(ii) 	Correct figure	**1**
	(iii) (a) In Δ ACQ $\dfrac{QC}{AC} = \tan 60° = \sqrt{3}$ QC = 240 m Height of statue including base = 240 m Height of statue excluding base = 240 – 58 = 182 m **OR** (iii) (b) 		**1** **1**

QR = 240 – 40 = 200 m ½

In Δ QRB

$\dfrac{QR}{RB}$ = tan 30° = $\dfrac{1}{\sqrt{3}}$

Horizontal distance RB = 200 $\sqrt{3}$ m ½

Correct figure ½

In Δ PRB

tan α = $\dfrac{PR}{BR}$

$\quad$ = $\dfrac{18}{200\sqrt{3}}$ or $\dfrac{3\sqrt{3}}{100}$ ½